Physics 116 Lab Manual

Amherst College
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General Instructions

Laboratory work is an integral part of the learning process in the physical sciences. Reading textbooks and doing problem sets are all very well, but there's nothing like hands-on experience to truly understand physics. The laboratory sessions complement your classwork. If you mentally dissociate the two and view the labs as something to be ticked off a list, you are doing yourself a great disservice, missing out on an excellent opportunity to learn more deeply.

In addition to course specific objectives, lab work is meant to develop analytic skills. Various factors may influence the outcome of an experiment, resulting in data differing noticeably from the theoretical predictions. A large part of experimental science is learning to control (when possible) and understand these outside influences. The key to any new advance based on experiment is to be able to draw *meaningful* conclusions from data that do not conform to the idealized predictions.

Skills

The laboratory sessions of Physics 116 are designed to help you become more familiar with basic physical concepts by actually carrying out quantitative measurements of physical phenomena. The labs attempt to develop several basic skills and several "higher-level" skills. The basic skills include:

1. Developing and using operational definitions to relate abstract concepts to observable quantities. For example, you'll learn to determine the acceleration of an object from easily measured quantities. One important facet of this skill is the ability to estimate and measure important physical quantities at various levels of precision.

2. Knowing and applying some generally useful measurement techniques for improving the reliability and precision of measurements, such as use of repeated measurements and applying comparison methods.

3. Being able to estimate the experimental uncertainties in quantities obtained from measurements.

The higher-level skills include the following:

1. Planning and preparing for measurements.

2. Executing and checking measurements intelligently.

3. Analyzing the results of measurements both numerically and, where applicable, graphically. This skill includes assessing experimental uncertainties and deviations from expected results to decide whether an experiment is in fact consistent with what the theory predicts.

4. Being able to describe, talk about, and write about physical measurements.

The laboratory work can be divided into three parts: 1) preparation, 2) execution, and 3) analysis. The preparation, of course, must be done before you come to you laboratory session. The execution and analysis (for the most part) will be done during the three-hour laboratory sessions.

Preparation

We *strongly* suggest that you do the following before coming to lab:

1. Read the laboratory instructions carefully. Make sure that you understand what the ultimate goal of the experiment is.

2. Review relevant concepts in the text and in lecture notes.

3. Outline the measurements to be made.

4. Understand how one goes from the measured quantities to the desired results.

5. Organize tables for recording data and the equations needed to relate measured quantities to the desired results.

Execution and Analysis

A permanently bound quadrille notebook is required for the lab, and one will be handed out at the beginning of the first lab. The cost of the notebook will be billed to your Amherst College account. The notebook is for recording your laboratory data, your analysis of them, and the conclusions you draw from them. The notebook is an informal record of your work, but it must be sufficiently neat and well organized so that both you and the instructor can understand exactly what you have done. It is also advantageous for your own professional development that you form the habit of keeping notes on your experimental work—notes sufficiently clear and complete that you can understand them much later. Developing a good note-taking technique requires consistent effort and discipline, skills that will be of great value in any professional career. If you become a research scientist, you will often (while writing reports or planning a new experiment) find yourself referring back to work you have done months or even years before; it is essential that your notes be sufficiently complete and unambiguous that you can understand exactly what you did then.[1] In the health professions, patient charts are analogous to the lab notebooks. Here, detailed notes are critical because many people will need to read and understand the information without the benefit of talking with the doctor who wrote it. In keeping a laboratory notebook, it is better to err on the side of verbosity and redundancy than to leave out possibly important details.

Appendix B in this lab manual gives instructions on how to keep a good lab notebook. You will be expected to adhere to these guidelines throughout the semester. In fact, we feel that keeping a good laboratory notebook is so important that we have decided to base part of your laboratory grade (equivalent to one formal lab report) on the quality of your lab notes. Your notebook will be evaluated at the end of (and possibly during) the semester.

You should have your lab notebook initialed by one of the instructors before you leave each lab session.

[1] There have been instances in which a researcher's notebooks have been subpoenaed or used as the basis for priority claims for patents.

Uncertainties

The stated result of any measurement is *incomplete* unless accompanied by the uncertainty in the measured quantity. By the uncertainty, we mean simply: How much greater, or smaller, than the stated value could the measured quantity have been before you could tell the difference with your measuring instruments? If, for instance, you measure the distance between two marks as 2.85 cm, and judge that you can estimate halves of mm (the finest gradations on your meter stick), you should report your results as 2.85 ± 0.05 cm. More details on uncertainties will be given in the notes on Experimental Uncertainties.

An important (if not the *most* important) part of the analysis of an experiment is an assessment of the agreement between the actual results of the experiment and the *expected* results of the experiment. The expected results might be based on theoretical calculations or the results obtained by other experiments. If you have correctly determined the experimental uncertainty for your results, you should expect your results to agree with the theoretical or previously determined results within the combined uncertainties. If your results do not agree with the expected results, you must determine why. Several common possibilities are the following:

1. You underestimated the experimental uncertainties.
2. There is an undetected "systematic error" in your measurement.
3. The theoretical calculation is in error.
4. The measurements are in error.
5. Some combination of the above.

Sometimes these deviations are "real" and indicate that something interesting has been discovered. In most cases (unfortunately), the explanation of the deviation is rather mundane (but nevertheless important). Remember that small deviations from expected results have led to several Nobel prizes.

Never erase data or calculations from your notebook. If you have a good reason to suspect some data (for example, you forgot to turn on a power supply in the system) or a calculation (you entered the wrong numbers into your calculator), simply draw a line through the data or calculation you wish to ignore and write a comment in the margin. It is surprising how often "wrong" data turn out to be useful after all.

Reports

You will prepare a report for each of the laboratory sessions. We will have two types: (1) short (informal) reports with an exit interview conducted by one of the laboratory instructors and (2) longer written (formal) reports.

Informal reports will, in general, focus on your in-class record of the experiment during the lab time along with your answers to the questions posed in the writeup for each lab. These short reports need not describe the entire experiment, however they should be complete and self-contained.

Formal reports will be required for three of the labs (see schedule). For formal reports, you are to prepare a somewhat longer, written account of your experimental work. These reports should include a complete description of the experiment and its results. They should be typed (use a word processor) on separate sheets of paper (not in your lab notebook) and are to be turned in one week later. You should pay special attention to the clarity and conciseness of your writing. If we find that your report would benefit from rewriting, we will ask you to submit a revised version of the report before a grade is assigned. Guidelines for preparation of formal lab reports are included in Appendix D. While you will work in groups when you collect and analyze data in the lab, *each lab partner will write his/her own, independent lab report.*

Grading

You must successfully complete all of the labs in order to pass Physics 116. We set the labs up only for the week they are to be performed, so if you have to miss a lab because of illness, family difficulties or other legitimate reasons, please let us know in advance (whenever possible), so we can arrange for a make-up time.

You will receive a grade for each of the formal lab reports. These grades along with an evaluation of your lab notebook (which will be weighted like one formal lab report) and an overall evaluation of your performance during the labs will constitute the lab portion of your course grade.

Intellectual Responsibility

Discussion and cooperation between lab partners is strongly encouraged and, indeed, often essential during the lab sessions. **However, each student must keep a separate record of the data, must do all calculations independently and must write an independent lab report.** It is strongly advised that students do not communicate with each other, in person or electronically, once the writing process has begun. Specific questions concerning the writing of reports should be directed to the instructor or teaching fellow. In addition, laboratory partners are expected to share equally in the collection of data. The sharing of drafts of reports, use of any data or calculations other than one's own, or the modeling of discussion or analysis after that found in another student's report, is considered a violation of the statement of Intellectual Responsibility.

We wish to emphasize that intellectual responsibility in lab work extends beyond simply not copying someone else's work to include the notion of scientific integrity, i.e. "respect for the data." By this we mean you should not alter, "fudge," or make up data just to have your results agree with some predetermined notions. Analysis of the data may occasionally cause you to question the validity of those data. It is always best to admit that your results do not turn out the way you had anticipated and to try to understand what went wrong. You should **never** erase data which appear to be wrong. It is perfectly legitimate to state that you are going to ignore some data in your final analysis if you have a justifiable reason to suspect a particular observation or calculation.

Laboratory Syllabus

Physics 116 meets Wednesday at 1:00–4:00pm, Thursday at 8:30–11:30am and Thursday at 1:00–4:00pm for the laboratory exercises. This semester we will begin and end promptly at the appointed hours. (Please plan to spend the entire period in class.) All of the exercises must be completed to receive a passing grade.

For the three sessions marked with a ★, below, a formal written report is due on or before the beginning of the following lab class.

Date	Lab
Jan. 25/26	Intro Lab Meeting
Feb. 1/2	1 Force Table
Feb. 8/9	2 Free Fall ★
Feb. 15/16	3 The Bouncing Ball
Feb. 22/23	No Lab (Exam I)
Feb. 29/March 1	No Lab
March 7/8	4 Acceleration on an Inclined Plane ★
March 14/15	5 The "Outward Force" due to Rotation
March 21/22	No Lab (Spring Break)
March 28/29	6 Conservation Laws in Collisions
April 4/5	No Lab (Exam II)
April 11/12	7 The Ballistic Pendulum
April 18/19	8 Simple Harmonic Motion ★
April 25/26	9 Standing Waves
May 2/3	No Lab

1 Force Table

Read this laboratory handout carefully before coming to lab. Also read Appendix B in the lab manual on how to keep a good notebook.

It's rather amazing that mathematical concepts, which after all are inventions of the human mind, should find such wide applicability in describing the physical world. In fact, there is no guarantee that a physical application will be found for every mathematical scheme. For example, there is nothing special about vector algebra that would suggest a priori that it would find wide use in the understanding of motion. But, in fact, it does turn out to provide a natural logic for comprehending both the kinematics and dynamics of the motion of objects in two or three dimensions.

The force table apparatus provides a simple means for testing another application of vector algebra, namely, that vector algebra provides a framework for describing the addition of forces. The face of the table is a heavy metal disk calibrated around the edge in degrees (360° total). A brass ring, held near the center of the table by a vertical pin, carries three attached strings, each of which can be directed over a pulley fixed to the table edge and terminated with a hanging weight pan. The addition of weights to these pans creates a tension in each string. Each string is thus said to pull on the brass ring. Such "pulls" constitute a simple introduction to the concept of forces.

Figure 1: Force Table

1.1 Experiment

Examine the force table example set up in the lab in order to appreciate what your goal will be. Notice that if the weights and string angles are adjusted just right, you can remove the restraining pin, and the ring is held in equilibrium by the action of the three string forces alone. However, there is nothing unique about the choices used in the example: there is an infinity of combinations of "string-directions-plus-weights" that will permit equilibrium of the ring. Your task is to show that for other combinations, your chosen values are explained by vector algebra.

1.2 Procedure

The procedure consists of three trials:

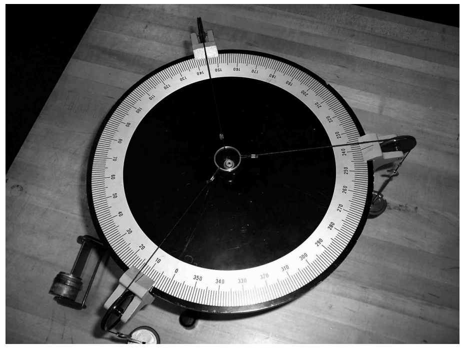

Figure 2: Top view of the force table

1. For the first trial, both partners should collaborate to experimentally determine a balance of forces.
   (a) Decide on weights and angles for two of the strings. Start with at least 100 grams on each of the two strings. Do not use bare hangers. These will remain fixed during the trial. If you wish, you may use "easy" angles like 0° and 90°, or 0° and 120°.
   (b) Without prior calculation, apply a weight to the third string and vary its magnitude and angle until the third string balances the forces exerted by the first two.
   (c) Once you have verified that the balance is "pretty good", estimate uncertainty by adding or subtracting weight until the ring has moved noticeably off-center. Also move the angle in both directions until the ring is off center.
   (d) Using the weights and angles for the first two strings, either numerically or graphically, calculate the third, balancing force on the ring.
2. For the second trial, you will calculate the theoretical magnitude and direction of the third force *before* you test to see if your calculations are consistent with experiment.
   (a) Choose two new weights set at new angles. Do not separate the first two strings by "easy" angles like 90°. The strings should also be weighted with unequal masses.
   (b) First, conduct a rough graphical analysis (using a ruler and protractor) to calculate the third force with a scale drawing.
   (c) Next, conduct a numerical analysis (using trigonometry). To guide your application of vector algebra to this situation, we recommend that you mentally impose an XY-coordinate frame on the table face with the origin at the table center, and one axis aligned with the zero degree marker.
3. For the third trial, choose three angles and keep them fixed (again, do not use "easy" angles). Next, choose two arbitrary weights for two of the strings. Try to balance the forces. Your goal should be to get the forces balanced by varying the fewest number of quantities (weights and angles). First try varying only the weight on the third string. If that doesn't work, consider

whether fixing the third weight and varying only the angle would work. You may want to consult with an instructor for advice. Once you have it balanced, confirm that the graphical and numerical analysis methods produce a total force of zero on the center ring.

1.3 Results

The experimental results and those from the two theoretical methods should now be compared and any differences reconciled. Thus for each trial you would end up with results something like this for the 3rd force:

	Magnitude (grams)	Direction (degrees)
Theory (graph)	230	120
Theory (numerical)	231	119
Experiment	205 ± 2	129 ± 2

Note that in this case there is not good agreement between the predicted and measured results, and some further investigation is needed (or perhaps vector algebra does not apply to your forces...).

1.4 Exit Interview

The report for this lab is to be an informal report in the form of an exit interview. Be prepared to give an oral presentation of the the work you have done. You will be asked a series of questions covering all parts of the experiment: theoretical background, experimental technique, analysis and results. Your lab notebook will be reviewed during the exit interview so make sure you have detailed notes and all the relevant information (tables, figures, plots, etc.) in your notebook. Please refer to appendix B for information on keeping a notebook.

2 Free Fall ★

In this laboratory we will investigate the motion of a freely falling object in the Earth's gravitational field. In particular, we wish determine whether such an object experiences a constant acceleration. In addition, if the acceleration of a falling object is constant, we would like to determine the magnitude of this acceleration.

2.1 Apparatus

To study this process, we'll measure the position of a falling object as a function of time. Each lab station has a weight that can be attached to one end of a piece of paper tape. The tape passes through a small timing module (see Figure 3). The module contains a small oscillating hammer (not visible) that 40 times/sec strikes a sandwich made up of a piece of carbon paper, the tape, and a strike plate. As the tape passes through the module, a series of dots is recorded on the tape. From these dots, you can determine how the position of the falling mass varies with time.

Figure 3: the PASCO tape timer.

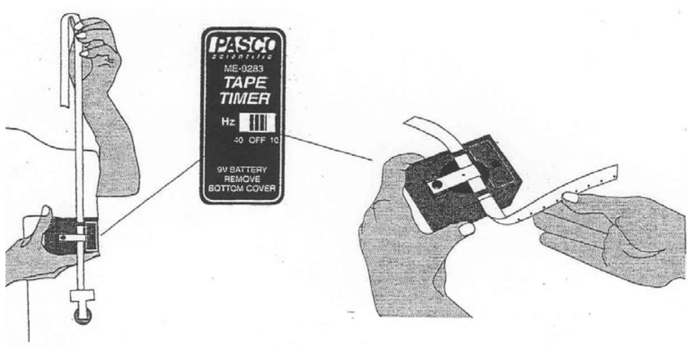

Figure 1

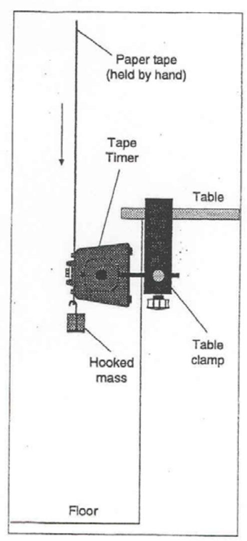

Figure 4: Using the PASCO timer.

2.2 Experiment

1. *Set up and collect data*

   Set up the apparatus as shown in Figure 4. The tape should be ∼1.5 m long, and the bucket of sand should be positioned so that the falling weight will hit it rather than the floor. Hold the tape *vertically* above the timer module and turn on the module to 40 Hz (40 cycles/sec). Make certain that the mass is not swinging, then release the tape.

   > **Each student should produce a tape to be analyzed. You should work together on the analysis, but your tape should be the basis for your lab report.**

2. *Choose the zero point*

   Choose a well-defined dot just beyond the confusion of dots at the beginning of the tape. Let this first good dot on your strip be the origin ($s = 0$) of a position-coordinate scale. (We call the position coordinate variable s to avoid confusion with the quantities x and y that you will use in attempting to find a straight-line plot later on.) Also let the instant of time at which the body was at $s = 0$ be $t = 0$. It is crucial to note that the velocity at $t = 0$ is not zero.

3. *Record positions and times*

   Using either a meter stick or a 30 cm ruler (whichever you prefer) measure the positions of the dots. Think about how to position the meter stick to avoid "parallax" problems. Record a table of values of s versus t in your lab notebook. Use the full length of the tape.

2.3 Analysis

The main question to ask is "Did this motion take place with constant acceleration?" You will recall from class and your text that one-dimensional motion with a constant acceleration (a) may be described by the expression

$$s = v_0 t + \frac{1}{2} a t^2 \tag{1}$$

Here, v_0 is the velocity of the object at $t = 0$ and s is the coordinate position *assuming that* $s = 0$ at $t = 0$. We would like to know if a freely falling object experiences a constant acceleration. If it does, then its motion must be described by Eq. 1. To test this hypothesis, you might try to make a plot of your experimental points s versus t and see if it is compatible with Eq. 1. Such a plot would look something like Figure 5.

It would be difficult to judge from simply looking at this plot if the motion is consistent with a parabola (Eq. 1). Without computerized curve-fitting, it would be even more difficult to determine the acceleration of your falling object from such a plot. We can, however, be clever and rearrange Eq. 1 so that by plotting an appropriate combination of experimental parameters, we would expect to have a graph of a straight line. In addition to enabling you to test the general dependence anticipated in Eq. 1, this method allows you to determine both the acceleration a and the velocity v_0 your object had at $t = 0$, from the slope and intercept of the resulting line respectively.

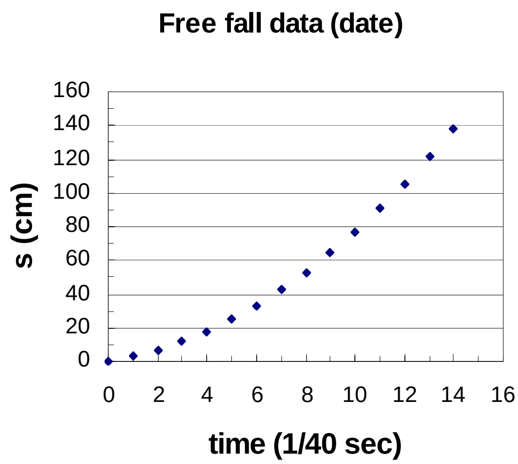

Figure 5: Uniformly accelerated motion
Displacement versus time.

One way of rearranging Eq. 1 to achieve these goals is to divide it through by t. We then find

$$\frac{s}{t} = \frac{1}{2}at + v_0. \tag{2}$$

Recall that the equation describing a straight line of slope m and intercept b is

$$y = mx + b. \tag{3}$$

So, if you make a plot of your data with $y = s/t$ and $x = t$, then you would expect to find a straight line with slope $a/2$ and intercept v_0, provided Eq. 1 correctly describes the motion.

1. *Enter your data into Microsoft Excel*

   Each station will have a netbook with Microsoft Excel. Additional computers are located in Merrill 215. If you are unfamiliar with Excel, we strongly recommend that you read the Accelerated Excel manual. In a new spreadsheet, enter your values of t in the first column. Be sure to label the column with the quantity it contains (time) and the units. In the adjacent column, enter your values for s.

2. *Create a column for s/t*

   In an empty column, select the cell in the row corresponding to $t = 1/40$ s. We will now enter a formula to calculate the values of s/t automatically. To let Excel know you are entering a formula, you must begin your entry with an "`=`" sign. Type "`=`", then click on the cell in the same row containing s. The formula bar should show "`=`", followed by a letter and a number, corresponding to the column and row of the cell you just clicked on. To tell Excel that you want to divide this value by t, type "`/`", click on the cell in the same row containing t, and press `Enter`. Excel should calculate s/t for $t = 1/40$ s.

Copy this formula into every cell in the column for which s and t are defined.

Note: If you try to copy this formula into the $t = 0$ row, Excel will return a `#DEV/0!` error. Because 0/0 is undefined; you must exclude this point from your data analysis.

3. *Graph and analyze your data*

   Make a graph of s/t vs. t. Let us assume that your data are consistent with a constant acceleration for your freely-falling mass. There are a number of methods that could be used to estimate the uncertainty on that acceleration. One method would be to repeat the experiment a number of times and to find a mean value for the acceleration and also a standard deviation from this mean value (see appendix E). We shall not expect you to carry out numerous repetitions of the experiment. However, we shall expect you to repeat the experiment at least once, and to analyze and graph both sets of data. In addition, you'll be expected to use the spreadsheet to determine a "best" value for both v_0 and the acceleration a.

4. *Determine the uncertainty in v_0 and a*

   In the "Tools" menu, select "Data Analysis"[2], and select "Regression" in the data analysis window. You will be presented with a dialog containing a number of options; for your purposes you only need enter an X range and a Y range before clicking OK. Excel will then produce a "SUMMARY OUTPUT" page. The important numbers on this page are in the "Intercept" (y_0) and "X Variable" (slope) rows, under "Coefficients" ("best" value) and "Standard Error" (uncertainty).

5. *Repeat for each lab partner*

   Obtain another set of data, and repeat the analysis above, for each member of the lab group. Each person should have his or her own data tape and plot of s/t versus t.

Note: If any of your experimental points are clearly inconsistent with the rest of the data, recheck your measurements and repeat the spreadsheet analysis.

Please **PRINT** out your Excel plots and analysis and tape them into your notebook in the analysis section.
Q: Do your values of the acceleration a agree, within their uncertainties? What about your values of the initial speed v_0? Why or why not?
Q: How does the acceleration a depend on the the mass of the weight?

2.4 Formal Lab Report

The report for this lab is to be a formal report giving a complete description of your lab work. You should assume that the reader has a physics background equivalent to Physics 116, *but knows nothing about what you did in the lab.* Your report should be sufficiently complete so that the reader will know exactly what you did and can understand the significance of your results. The format for a formal lab report is detailed in Appendix D. Also, be sure to include your paper tape with your report.

[2]For some reason, "Data Analysis" is only available in Excel 2007 while you have selected some portion of the spreadsheet: if you can't find "Data Analysis" in the data menu, you probably have a graph selected.

3 The Bouncing Ball

3.1 Introduction

The experiment for this week could not be simpler, at least conceptually. You'll have a superball and will be expected to analyze a part of its motion after it is dropped from a height of about 80 cm. Because the experiment itself is so modest, we've decided to use it to give you a taste of automated data acquisition and also an opportunity to use a computer for data analysis. In the actual experiment, you'll use a motion sensor, connected to a laptop computer. Once triggered, the sensor sends out a specified number of pulses (e.g. 100) of high frequency sound with a specified time interval between pulses (e.g. 0.05 seconds). (For the examples given, the pulses are emitted over a time period of 5 seconds). See Figure 6 for a graph of the emitted signal.

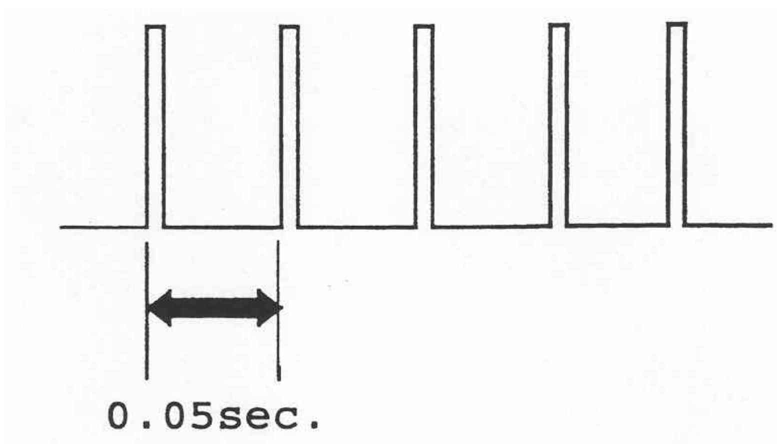

Figure 6: Pulses from the motion sensor

The pulses of sound travel at a speed of about 330 m/s and, in a carefully executed experiment, will be reflected back to the motion sensor by an object along the path of the pulses. If the object is not too far away, and is not moving too rapidly, a given pulse reflected by the object will arrive back at the sensor well before the next pulse is sent out. The motion sensor (in conjunction with the computer) measures the time interval between the emission of a given pulse and the reception of its reflection. From this time difference and the known speed of sound, it calculates the position of the moving object. The sensor-computer assembly can also calculate the time at which a given pulse was reflected from the moving object. The values of time and object position (relative to the sensor) are stored on the computer. The motion sensors work most reliably between about 0.5 m and 1.5 m.

3.2 Preliminary Experiment

Drop the ball from a height of about 80 cm above the floor and observe its subsequent motion. Try to keep the motion more or less one dimensional for several bounces. Make a qualitative position-time graph of the motion in your lab book, assuming the motion to be one dimensional. Be sure to

indicate whether you have chosen up or down as the positive direction.
Q1: Which points on the graph correspond to its collision with the floor?
Q2: Which points correspond to the top of a bounce?

3.3 Experiment

1. *Practice your technique before recording data*

   Center the ball directly under the motion sensor at a height of about 80 cm above the floor and at least 20 cm below the motion sensor. One of you should lightly hold the ball and then release it from rest. Practice this several times until the ball initially hits a point near the center of the floor tile directly beneath the sensor and then bounces at least 2 or 3 times within the area of the tile. After releasing the ball you should quickly move your hands away.

2. *Acquire data*

   Once you have a technique for releasing the ball, you are ready to acquire data. Using the motion sensor + computer, record the position of the ball every 0.05 seconds (20 Hz sample rate) over an interval of 5 seconds. This should incorporate a few bounces of the ball. See Appendix A for instructions on the use of the motion sensor + computer system.

3. *Check your data*

   Examine a graph of the data on the computer. If the ball has not had several bounces within the bounds of the tile, then the display will be very noisy. In that case, repeat your experiment until you get clean data.

4. *Think about your data*

   Examine the part of the display that contains two or more "good" bounces.

   Q3: How does the display compare with your sketch of the results from the preliminary experiment? Comment on any differences.

Once you have recorded satisfactory data, export them to a .txt file (see Appendix A for details).

3.3.1 Data Analysis

1. Open the data of time (t) and position (y) in a blank spreadsheet in Excel (see Appendix A for details). Be sure to label your columns and indicate units.

2. Make a graph of these data. Note the approximate times corresponding to one good bounce (one parabola).

3. In an empty column next to the data, calculate the average or *mean* time between two consecutive times. Do this for each of the times recorded.

4. Now, in the next empty column, again select the cell that corresponds to the first point in the data. Enter into this cell a formula that calculates the average velocity between the first and second point. Fill the other relevant cells in the column with this formula.

5. Make a graph of average velocity vs. mean time.
   Q4: Why must we plot average velocity against *mean* time?
   Q5: Do you think it is reasonable to regard this graph as a graph of instantaneous velocity versus time? Explain your answer briefly.
   Q6: What can you say about the acceleration of your bouncing ball?

6. Locate the first and last points on the best-looking bounce; this is usually easier to judge from the velocity vs. time graph than from the position vs. time graph.

7. Generate a new graph of average velocity vs. mean time from a subset of points that represent one bounce.

8. Use Excel `Data Analysis` > `Regression` to obtain the slope and intercept of the best-fit straight line that can be passed through the subset of data points. Obtain this slope and intercept for the best fit straight line passing through the graph you have produced.
   Q7: What value for the slope do you get, and what is its statistical uncertainty? Comment upon this result.

9. Analyze both halves of the parabola separately to determine the individual accelerations of the ball on the up and down parts of the bounce. Do your results give you any evidence for the effects of air drag?

10. Label the columns of your spreadsheet (including units), and the axes of your x vs t and v vs t graphs, if you've not already done so. Print out the graphs.

11. Write a summary of your results in your lab notebook. Make certain you have answered all the questions. Include your printouts (staple, tape or glue).

4 Acceleration on an Inclined Plane ★

Read this entire section before coming to and beginning the lab. You will run this experiment twice, for two inclination angles. Also, read Appendix E (Experimental Uncertainty Analysis) before coming to the lab.

One of Galileo's great contributions to experimental science was his use of the inclined plane as a means of "diluting" gravity; that is, as a way to slow down free fall so that precise measurements of the motion could be made. A modern refinement of Galileo's rolling-ball inclined plane is the air track. On the air track, gliders are supported by a thin layer of air and move nearly frictionlessly along the track.

Your job is to repeat Galileo's inclined plane experiment of measuring $g = 9.81$ m/s^2 to an accuracy of ±1%. [You may not be able to achieve this accuracy, but this is what you will try for. A very rough rule of thumb in physics is that a 10% measurement of something is fairly easy; a 1% measurement requires considerable thought and care, and a 0.1% measurement is apt to be extremely difficult.]

4.1 Theoretical Background

Assume that you know the following:

1. If frictional effects are completely negligible, a glider should move down a tilted airtrack with *constant* acceleration, a. [Frictional effects include not only possible friction between the glider and the track, but also drag due to air resistance.]

2. Without friction, the numerical value of a would be given by

$$a = g \sin\theta = g\frac{h}{L}, \tag{4}$$

   where $g = 9.81\ \mathrm{m\,s^{-2}}$, and L and h are simply geometrical quantities in a right triangle describing the tilt of the track as shown in Figure 7.

   The point here is that if we measure a, we can then compute g from Equation 4.

3. We assume that the kinematic relations for the case of constant acceleration are applicable to the motion of the glider on the airtrack. In particular, the coordinate position s of the glider as a function of time, t, on the track is described by

$$s = s_0 + v_0 t + \frac{1}{2}at^2. \tag{5}$$

where s_0 is the position at $t = 0$ and v_0 is the velocity at $t = 0$.

Important Note: It is essential that you be prepared to calculate quickly the experimental values of g from your observations, so that you can see how the results are coming out and be ready to modify procedures if necessary. [It is always a good idea to do preliminary analyses of at least some of the data while in the lab, if possible. It may turn out that either you have forgotten to take one vital measurement, or that the results are very peculiar and should be repeated, perhaps with a change in technique. This is just as true in a "real" experiment as in an introductory course.]

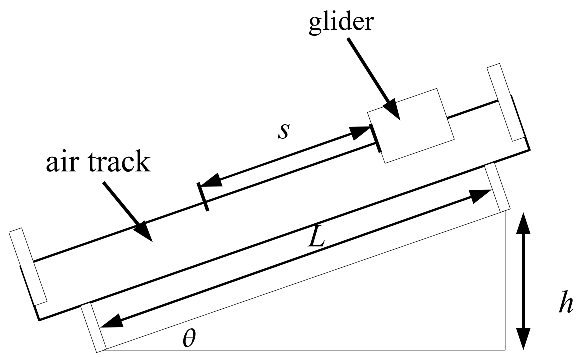

Figure 7: Definition of the parameters involved in the inclined plane. Note that L and the value of $s - s_0$ are NOT the same.

4.2 Preliminary Procedure

1. *Level the track and determine the uncertainty*

   Level the track by adjusting one of the leveling screws. Adjust only the screw at the end where there is a single one; if you fiddle with the two at the other end, you will change the side-to-side leveling which has already been adjusted. You will want to know how accurately you can level the track. Here's one way of doing that: with the track "level," put a file card under the "foot" at one end. Does that definitely make it *not* level? Or do you need 2 or 3 or 4 cards? Now remove the cards and find out how thick a file card is. Use a Vernier Caliper for this measurement. These tests should give you a good idea of the uncertainty in h.

2. *Measurement of a*

   Put the thicker aluminum riser block under the foot of the air track. Release the glider from rest and time (using a stopwatch) how long it takes to travel a chosen distance $s - s_0$ (see Fig 7). Use Eq. 5 to calculate a. Repeat 4 more times. Average the five trials to get the best value of a. The uncertainty on this value is given by the spread of your values (ideally it is the standard deviation as defined in Appendix E. A rough estimate is given by half the difference between your maximum and minimum values).

3. *Best estimate of g*

   Now using your best values of h, L, a and Eq. 4, calculate the average of your values for g. This is your best estimate. Use the uncertainties of h, L, a and the Simplified Uncertainty Rules in Appendix E to determine the uncertainty of g.

   The agreement with the standard value for g ought to be reasonably good. If you are off by 10% or more, you probably made a mistake somewhere in your measurements or calculations. Check them over.

Q1: What is the precision of your estimate? What is the accuracy of your estimate?

4.3 Motion Sensor Experiment

We now want you to perform an experiment that will give you the best possible accuracy for g. There are two main reasons why your preliminary measurement of g may be inaccurate. You must consider how to eliminate or at least limit their impact.

4.3.1 Main problems to consider

Timing accuracy It is clearly vital that the stopwatch be started and stopped "correctly." If the watch is to be started upon release of the glider, and stopped when it travels $s - s_0$, these two events are qualitatively quite different: the start is rather slow, while the stop is quite fast. Thus, "reaction time" errors may not cancel out.

Air drag Air drag may be significant. For a glider going *downhill*, air drag would tend to slow the glider down and to make the apparent value of g *smaller* than expected. To correct for this, analyze a complete trip up and back down the track. Why does this cancel (to first order) the effects of air friction?

4.3.2 Performing the experiment

For your new experiment, initially use a $1''$ riser block.

1. *Automate the timing technique*

   Use the motion sensor instead of a stopwatch. The motion sensor should be set to take 100 readings at 0.2 sec intervals (5 Hz). The motion sensor should be situated ~ 150 cm along the airtrack, and its height should be set so that its center is 6 cm above the track.

   CAUTION: make certain that the glider on the airtrack never collides with the motion sensor!

2. *Start the glider moving and record data*

   Decide from where your glider will start and during which part of its motion you will record data. Remember that you are trying to minimize the impact of the two problems outlined above.

3. *Check your data quality*

   If you get a good quality position vs. time graph, you're ready to analyze the data. If your position-time graph is "noisy", try again. Make adjustments to the position and angle of the sensor, or the release point of the glider to eliminate the noise. If you can't eliminate the noise, see one of the instructors.

4. *Analyze your data using DataStudio*

   In DataStudio, be sure to select the velocity option (in addition to position) under the `Setup` menu. To display position and velocity on the same graph, drag the corresponding Runs from the `Data` column on the left onto the graph area. See Appendix A for more information on using DataStudio.

DataStudio can apply a quadratic fit to data of the form $At^2 + Bt + C$. Using the graph of position vs. time, select a subset of data representing one trip up and down the track (you can do this simply by dragging a box around data points). Then, choose `Quadratic Fit` from the `Fit` drop-down menu. Using the resulting coefficients and Eq. 5, obtain a value of a from your analysis. Also, obtain a value for the statistical uncertainty of a.

Next, look at the graph of velocity vs. time. Using approximately the same subset of points, apply a linear fit (choose `Linear Fit` from the `Fit` menu) and again determine a and the uncertainty of a.

For each value of a, calculate g and the uncertainty of g.

Q2: Do your values of g agree with each other? Do they agree with your estimate?

5. *Calculate the experimental uncertainty*

   Consider all sources of uncertainty in each of the measured quantities that go into the determination of g (see Appendix E in this lab manual) to estimate an experimental uncertainty in g that is due to the apparatus and measurement technique.

6. *Repeat for each lab group member*

   Take a complete set of data for each member of your lab group.

7. *Repeat with a different track angle*

   Repeat your experiment using two stacked riser blocks, again acquiring one data set for each partner. You may need to change the motion sensor time interval for this set of measurements. Does your result for g improve? Why or why not?

4.4 Formal Lab Report

Follow the guidelines in Appendix D for the formal lab report. Your report should include a careful description of the two methods used (stopwatch and motion sensor). Each partner should independently analyze their respective data (i.e., in a group of two, you should analyze two motion sensor data sets and all five preliminary trials). Discuss which method gives a more accurate result and why. Note how your experiment and analysis help to compensate for possible effects of air drag. You should also describe the calculations leading to your value of g and include an explanation of how you arrived at the "uncertainty" in your result. Discuss the difference, if any, between your measured value of g and the accepted value, and what might cause a possible discrepancy between the two.

5 The "Outward Force" due to Rotation

WARNING: The rotating elements in this lab can be hazardous. Please take care to keep your hands, hair, and clothing away from the spinning apparatus.

Make sure to read and understand this handout in its entirety before coming to and beginning the lab. You will conduct this experiment three times, for three different angular velocities.

Consider a body in uniform circular motion as shown in Fig. 8. The body might be, for example, a ball on the end of a string. We know from kinematics that a body in uniform circular motion is accelerated: although the speed remains constant, the velocity vector is continually changing in direction. We also know that the acceleration is a vector directed toward the center of the circle with a magnitude of

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}, \tag{6}$$

where r is the radius of the circle, and T is the period of the circular motion.

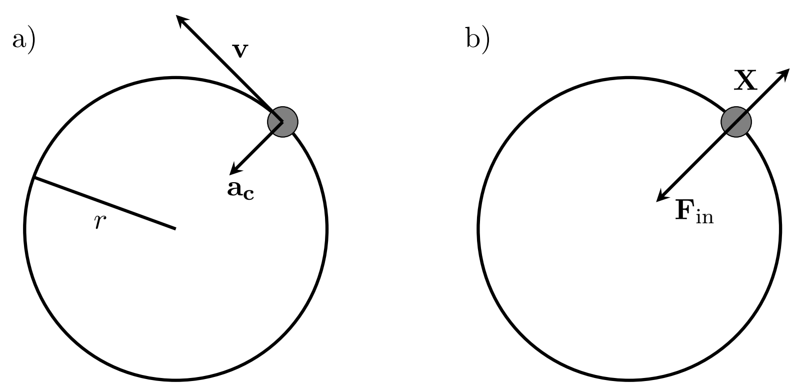

Figure 8: A body in circular motion (a) and a possible corresponding force diagram (b).

From our study of dynamics we know that the acceleration a body experiences can be predicted from a knowledge of the forces acting on it. What are the forces acting on the ball? The string is surely pulling on it; so, there is an inward force of magnitude F_{in} as shown in Fig. 8. However, one often finds references to a mysterious 'centrifugal force' of murky origin. To denote our skepticism, this force is labeled X in the diagram. You might expect the the net force on the object to be

$$F_{\text{net}} = F_{\text{in}} - X = ma_c. \tag{7}$$

You have been told in lecture that a careful accounting of forces, as measured in an inertial reference frame, does not require the invocation of 'centrifugal force' to explain an object's motion. If this is indeed true, then $X = 0$ and you should find that a plot of F_{net} vs. ma_c yields a slope of 1 with an intercept of 0. If not, the difference between F_{net} and ma_c is caused by the mysterious X force.

In this experiment, instead of using a ball on the end of a string we will use a cylindrical mass attached to a spring inside a rotating carriage. The only property of the spring we need to know is that, by Hooke's law, the force exerted by the spring (that is, F_{in} in Fig. 8b) depends only on how much the spring is stretched. Thus F_{in} can be measured (using the apparatus described below) by a separate experiment in which the spring is stretched by the same amount as under running conditions.

5.1 The Apparatus

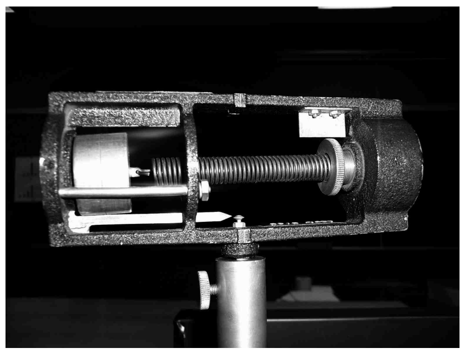

Figure 9: Side view of the apparatus.

The carriage consists of a mass (m) which is attached to a spring but is otherwise free to slide within a cylinder whose axis is horizontal (see Fig. 9). The entire device is rotated about a vertical axis at an adjustable speed. For any particular (constant) speed, the period of rotation (T) is constant, and the mass rotates at a constant distance r from the axis, corresponding to uniform circular motion at a radius r with a period T.

5.2 The Experiment

1. *Assemble the apparatus*

   There is a thumbscrew attached to the spring in the carriage; begin with this screwed in as far as possible. Extending from the carriage is an 8 mm diameter cylindrical pin with one flat face. Mount the carriage on the rotator by inserting this pin into the rotator shaft and tightening the thumbscrew. Be sure that the locking thumbscrew presses against the flat face of the pin.

2. *Adjust the rotation rate and measure the period (T)*

   The rotation rate can be controlled by the experimenter by an adjustment screw, and the period T can be measured with a stopwatch and a counter by counting the number of revolutions occurring during a set time period. (Use a fairly large total time in order to minimize the effect of error due to your reaction time in starting and stopping the stopwatch.) To avoid the problem of trying to determine r while the apparatus is in motion, a pointer device is built into the apparatus. You can adjust the rotation so that the mass assumes the value of r needed to **just activate** the pointer. (This device is difficult to describe. You will need to carefully observe the apparatus in order to figure out how it works. Its operation will then become clear.)

You will make three measurements at three different rotation rates:

i. Measure the "best" rotation rate, at which the mass barely activates the pointer.

ii. To get an upper bound, find the *lowest* rotation rate at which the pointer is consistently in the "up" position. Note that rotation rates higher than this will also give a consistent "up" position because the mass becomes pinned against the end of the carriage.

iii. Place a lower bound by finding the *highest* rotation rate at which the pointer is consistently in the "down" position.

3. *Measure r*

   To measure this particular value of r, you can use another gadget with which you can extend the spring under static conditions until the pointer is just activated. Then use calipers to measure the distance from the axis of rotation to the center of mass. Since this mass is not a point mass, the manufacturer has obligingly marked a line on it to indicate its center.

4. *Calculate a_c and ma_c*

   From the measurements described above, compute the magnitude of the centripetal acceleration, v^2/r, and compare it to the familiar acceleration due to gravity, g, to get a feel for how large a_c is in this particular rotating system.

5. *Determine F_{in}*

   To determine the magnitude of the force F_{in} exerted by the spring when m is in the critical position that actuates the pointer, you can suspend the carriage vertically and attach hanging weights until m is pulled down the appropriate distance. Be sure to determine an uncertainty for this quantity as well.

6. *Repeat the experiment for two other spring tensions*

   Repeat the experiment with the thumbscrew attached to the spring at its maximum extension, and again with the screw halfway between maximum and minimum extension.

5.3 Analysis

Make a plot of F_{in} vs. ma_c. Be sure to include error bars that account for the combined uncertainties of all variables used to calculate F_{in} and ma_c. Is your data consistent with the equation $F_{\text{in}} = ma_c$, or do you find evidence for a nonzero X force?

6 Conservation Laws in Collisions

Read the instructions completely and carefully before coming to lab. Also, work out how to calculate the uncertainty in the kinetic energy (K_f, K_i) and the momentum (p_f, p_i) for both elastic and inelastic collisions.

In this experiment we will use two conservation laws, the conservation of linear momentum and the conservation of mechanical energy, to study two types of collisions:

- Perfectly **inelastic** collisions. The two colliding bodies adhere to each other upon contact and move off as a unit after the collision.
- The ideal non-sticking, bouncy collision is called **elastic**. Our bouncy collisions will be nearly perfectly elastic.

6.1 Procedure

1. *Prepare the equipment: level the airtrack and test PASCO Motion Sensors*

   Start by leveling your air track. Set up *two* PASCO motion sensors positioned ∼140 cm apart, *facing* each other. The sensors should be centered over the airtrack. The sensors should be ∼6 cm above the track. Configure both sensors to record position vs. time.

   > **Do not allow the gliders to crash into the sensors. The sensors can easily be damaged. Make sure the gliders move freely under the sensors.**

2. *Determine the masses of the gliders*

   The air-track gliders have <u>approximately</u> the following masses:

   small glider 200 grams
   large glider 400 grams

   You should check the masses of the carts you use on the electronic balance.

3. *Test the motion sensors with a single glider*

   Push one glider along the length of the airtrack and record its position using both motion sensors. Remember, the sensors measure distances relative to themselves. Look at the graph of position vs. time. Based on your qualitative inspection of the graph, what can you say about the velocity of the single glider? Can you explain any differences in the two plots from the two sensors?

4. *Two body collision*

   Now measure the positions of the two gliders colliding with each other. Start with one glider at rest, between the sensors. Push the second glider into the first. Be sure to stop timing once one of the gliders is out of range or passes under a sensor. Look at the graph of position vs. time for the two gliders. Can you identify the moment of collision?

5. *Record position vs. time for the two gliders for at least 3 cases of collisions*

   For both elastic and inelastic collisions, you will investigate at least three cases (not necessarily in the order given below). The minimum three cases are:

   (a) small or large cart into another stationary cart of equal mass

   (b) small cart into large cart at rest

   (c) small or large cart into moving cart of unequal mass

6.2 Analysis

1. *Determine the velocites of the gliders*

   For each set of position vs. time data there are 4 distinct line segments which represent the 4 velocities of the gliders (2 before collision velocities and 2 after collision velocities). If the line segments are in fact lines, what can you say about the velocities? Use a linear fit (in DataStudio) on each of the line segments to determine the velocities of the gliders before and after the collision.

   Important: keep in mind that one of the sensors is looking *backwards*. How does this affect the velocity it measures? How will you correct the velocity?

2. *Calculate the momenta and kinetic energies*

   Now that you have the velocities and masses of the gliders, calculate the initial total kinetic energy (K_i), the final total kinetic energy (K_f), the initial linear momentum (p_i), and the final linear momentum (p_f).

   For each of the 6 total collisions, compare your momenta and kinetic energies. Is momentum conserved? Is kinetic energy conserved?

3. *Calculate sample uncertainties*

   Perform uncertainty calculations for just one of the collisions. Find the uncertainty in each of K_i, K_f, p_i, and p_f.

7 The Ballistic Pendulum

Make sure to read and understand this handout in its entirety before coming to and beginning the lab. Work out the derivations needed to complete the calculations in §7.1.

The ballistic pendulum (See Fig. 10 below.) was, once upon a time (before the era of high-speed electronics), used to measure the speeds of bullets. The method used to determine the speed of the bullet depends on the clever use of fundamental conservation laws in physics.

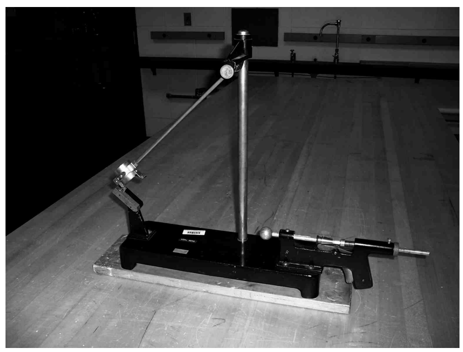

Figure 10: Ballistic Pendulum

A bullet, with mass m and initial speed v, collides with and becomes lodged in a mass M. (See Fig. 11 below.) The larger mass M is supported as a pendulum. The collision itself takes place so rapidly that m and M together can be considered as a closed system during the collision, i.e. as two particles which interact only with each other.

Immediately after the collision, the new composite object $(m + M)$ has a velocity to the right. Then $(m + M)$ acts as a pendulum and swings upward, trading kinetic energy for gravitational potential energy. The amount of potential energy gained is given by $(m + M)gh$, where h is the height through which the center of mass of $(m + M)$ rises. In this lab you will calculate the muzzle velocity v (or, more properly, the muzzle speed) of the bullet using a set of measurements of h. Using this information, you will then predict the range of the bullet (if it were not intercepted by the ballistic pendulum) and you will dramatically test your prediction. Before class, you should

- by considering the conservation laws which apply *after* the collision, derive an expression for v', the speed of the composite object just after the collision, in terms of h and the masses.
- by considering the conservation laws which apply *during* the collision, relate v' to the bullet's muzzle velocity v. Together with the previous expression, this gives an equation for v in terms of h.
- using this value of v, you can *predict* how far the bullet would travel if it didn't hit the pendulum and actually continued unimpeded until it struck the floor a distance H below.

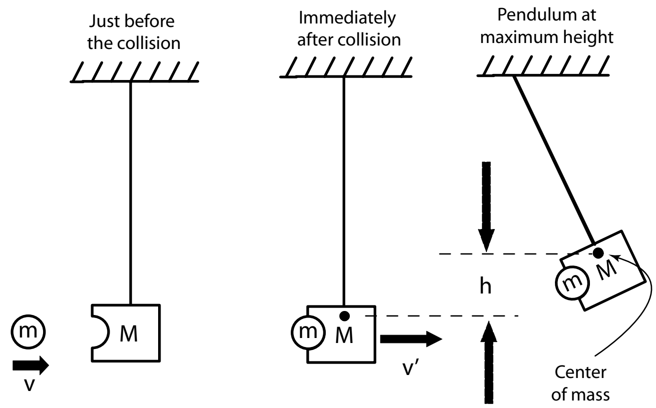

Figure 11: Ballistic Pendulum before and after collision

7.1 Procedure

Predict the horizontal distance the bullet will travel before striking the floor. Start by doing the derivation outlined below using only variables (before coming to lab), then measure values for the various parameters. Once you have a predicted range and its uncertainty, test your prediction.

1. *Measure h and its uncertainty*

   Be careful to measure h properly. For a point mass, the change in the gravitational potential energy is $\Delta U = mgh$. For an extended object, this same expression holds if h is the change in vertical position of the **center of mass**.

   There is a set of notches to catch the pendulum at approximately its highest point. The notches are numbered but the numbers are simply labels. You must measure the appropriate value of h directly.

   Obviously, you will want to make a series of measurements of h (at least 7) in order to determine its uncertainty.

2. *Weigh the projectile (m) and the pendulum bob (M)*

   The bullet can be weighed. Make sure you keep track of your bullet as its mass (m) is important and the bullets don't all have the same mass.

   The larger mass M can also be removed from the pendulum for weighing.

3. *Calculate the horizontal range of* **your** *projectile*

   Don't forget to calculate the uncertainty!

4. *Mark your prediction*

   Tape paper and carbon paper to the floor so that the bullet makes a distinct mark. Mark the expected impact point and the uncertainty range.

5. *Test your prediction*

   Be careful! The bullets move at fairly high speeds.

   Summon one or more of the instructors and let us see how good your prediction is. When testing your prediction, do not be confused by bounces!

   Carry out a series of trials (at least 5) of the horizontal range of the bullet as a projectile. Record the *actual* distance the bullet travels for each trial, and compare with prediction.

8 Simple Harmonic Motion ★

Make sure to read this lab in its entirety before coming to and beginning the lab. Before coming to lab, you must do the following exercises. 1) Substitute Eq. 10 into Eq. 9 (you will need to evaluate the second derivative of Eq. 10) and verify that Eq. 10 provides a solution to Eq. 9. 2) Find a relationship that expresses the period, T, as a function of the mass, m, and the spring constant, k. 3) Substitute Eq. 14 into Eq. 13 and show that Eq. 14 is a solution of Eq. 13 and from this determine the relationship between ω, g and L.

In this experiment you will explore two situations which give rise to simple harmonic motion (SHM). The common characteristic of both situations is that an object is acted on by a force which increases **linearly** as the object moves away from an equilibrium position:

$$F_x = -kx, \tag{8}$$

where x is the displacement from the equilibrium ($F_x = 0$) position. Such a force is called a restoring force since the force tends to pull the object back to the equilibrium position. The farther the object is away from equilibrium, the larger the force pulling it back.

The motion of an object acted on by such a force is described by a differential equation of the form

$$\frac{d^2x}{dt^2} = -\omega^2 x, \tag{9}$$

where, as above, x is the displacement of the object from its equilibrium position, and ω is a parameter called the angular frequency. The solution of this differential equation, assuming that x is at its maximum value at $t = 0$, is

$$x(t) = A\cos(\omega t), \tag{10}$$

where the amplitude, A, is the maximum displacement of the system from its equilibrium position. An important property of simple harmonic motion is that ω is independent of A. The period of the oscillation is given by

$$T = \frac{2\pi}{\omega}. \tag{11}$$

In this laboratory you will be investigating the extent to which the oscillatory motion of a pendulum and of a mass on a spring can be described as simple harmonic motion.

8.1 Mass on a Spring

A spring provides one of the simplest examples of a linear restoring force. The simplest case occurs when the motion is horizontal and the mass slides on a horizontal frictionless plane. Having few such planes available, we will instead study the vertical motion of a mass suspended from a spring. The vertical position of the mass is an oscillatory function of the form

$$y(t) = y_0 + A\cos(\omega t) \tag{12}$$

This is similar to Eq. 10 except that the equilibrium position, y_0, is not zero.

8.2 Simple Pendulum

For small oscillations, the angular displacement of the pendulum, θ, is described by the equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta. \tag{13}$$

Note that Eq. 13 has exactly the same form as Eq. 9. Hence, we may immediately conclude that the angular displacement described by $\theta(t)$ will execute simple harmonic motion

$$\theta(t) = \theta_0 \cos(\omega t) \tag{14}$$

where

$$\omega^2 = \frac{g}{L}. \tag{15}$$

8.3 Experimental Procedures

8.3.1 Mass on a Spring

1. *Study and measure the period*

   Choose one of the two conical masses provided and attach it to the spring. Measure the mass. Note: it is important to record the mass of the spring itself. In all calculations involving the oscillating mass, use the *total* mass:

   $$m_{tot} = m_{hanging} + \frac{1}{3}m_{spring} \tag{16}$$

   Measure the period of oscillation.
   Q1: Why is it better to determine the time for 50 oscillations than for just 5?

   Carry out measurements for at least different 3 amplitudes to see if the period is independent of the amplitude of the motion as predicted. Make sure you stay in the limit of small oscillations (< 15 cm). Record the values of the amplitudes you use.

2. *Determine the spring constant*

   From your period measurements, determine the spring parameter k.

3. *Repeat for the second mass*

   Attach the second mass to the spring and measure the period of its motion. Determine the spring parameter k from these measurements.
   Q2: For the two masses you used above, what *should* the ratio of the periods be? Calculate the ratio of the measured periods, and compare it with the predicted ratio. Do they agree within uncertainty? If not, why not?

4. *Hooke's Law*

   To test Hooke's Law make a series of measurements of the stretch of the spring x vs. the applied force $F = mg$. Hang a mass hanger on the end of the spring. Starting with 100g, add

mass to the hanger and measure the length of the spring. Make several measurements between 100-800g. Plot F vs. x. Describe qualitatively the relation between the weight and length of the spring.
Q3: What range of values does the spring obey Hooke's Law?
Determine the spring constant k using regression analysis. Compare the value obtained with the values determined in steps 2 and 3.

8.3.2 Mass on a Spring with Motion Sensor

Set up a motion sensor to observe the position, velocity, and acceleration of an oscillating mass as a function of time. In the equilibrium position, the mass (about 450g) should be about 70 cm above the sensor. You may have to try more than one combination of sample interval and number of samples before finding a good one.

1. Use the motion sensor to measure the position, velocity, and acceleration of the mass for a ∼5 second interval after you have displaced the mass about 5 cm from its equilibrium position and carefully released it. Do not start the motion sensor until after you have released the mass and moved well away from the experimental apparatus. Make certain no one is leaning on the table during the measurement.

2. Look at the graph of position vs. time. If you see 3 or so cycles of a relatively noise-free sine curve, you are ready to analyze the data. If the data are noisy, repeat the measurement.

3. Using DataStudio, construct a graph which has plots of x, v, and a versus t. See Appendix A for instructions on displaying all three types of measurements on the same graph.

4. Include the graph with your **Formal Report**. Discuss the relative phases of the x vs. t, v vs. t, and a vs. t graphs.

5. By Newton's second law, F should be directly proportional to a. This gives us a convenient way to measure k, since $ma = -kx$, and the motion sensor gives us both x and a. Make a plot of a vs. x, which you should include in your formal report. From the slope of the best-fit line, find k. Does this value agree with your previous values?

8.3.3 Simple Pendulum

1. *Study the period of a simple pendulum*

   Choose one of the masses and make a simple pendulum of about 1 meter length. Determine the period of oscillation for small oscillation amplitudes ($< 20°$). Carry out at least three measurements to see if the period is independent of the amplitude. Record the values of the amplitudes you use. Now make a pendulum of the same length with the *second* mass and once again measure the period.
   Q4: How do your results compare with the theoretical dependence of period upon mass?

2. *Calculate g using a simple pendulum*

Determine g by measuring the period of the *longest* pendulum you can conveniently make. To do this, we will have set up a pendulum in the central stairwell from the fifth-floor landing to the bottom of the stairwell, 4+ floors below. Give some thought to exactly what length you need to measure; what are the end points?

8.4 Formal Lab Report

A formal lab report is to be written based upon your findings. Follow the guidelines for formal reports in appendix D. The report should include the following:

1. Discuss your results for the masses on the spring.
2. Discuss your results for the pendulum.
3. For both cases *you* must decide what is important in your results and how to present that information (tables, graphs, etc.). Be sure to include some discussion of how experimental uncertainties affect your results.

9 Standing Waves

9.1 Background

In this experiment, you will set up standing waves on a string by mechanically driving one end of it. You will first observe the phenomenon of resonance. Though observations of resonance will only be qualitative in this lab, some experience of when resonance occurs will be important for the quantitative measurements you will be making.

9.1.1 Properties of Waves

The physical properties that you will measure, such as wavelength and frequency, are characteristics of standing waves. The theory (covered in lectures) relates the wavelength and frequency of standing waves to the speed of propagation of waves in the medium (the string, in our example):

$$v = \lambda f \tag{17}$$

where v is the speed of the wave, λ is its wavelength, and f its frequency.

Theory also relates the speed of propagation of waves in a medium to the properties of the medium. Typically, the wave speed is the square root of an elastic property of the medium such as a tension or stress divided by an inertial property such as mass density. In the case of waves on a string, the speed is given by

$$v = \sqrt{\frac{F}{\mu}} \tag{18}$$

where F is the tension in the string, and μ is its mass per unit length. (We reserve the letter T for period, and hence don't use it for the tension in the string here). In the lab, you will choose three different values of F, and for each make measurements of the wavelengths of standing waves for at least five different resonant frequencies. For a given tension, your data will allow you to investigate Eq. 17. The calculated average value of the velocity for each of the different tensions will permit a test of Eq. 18.

9.1.2 Standing waves

When the string is held (more or less) fixed at the two ends, the standing wave patterns shown in Figure 12 are possible. The places where the wave disturbance is zero are called the nodes and the places where the disturbance is a maximum are called anti-nodes. One may think of a standing wave as the sum of two traveling waves, one traveling to the right and the other to the left. The distance between two successive nodes (or two successive anti-nodes) in a standing wave is half the wavelength of either of the traveling waves that comprise the standing wave. Clearly the fixed ends of the string are always nodes. With that constraint, the longest wavelength standing wave corresponds to having an *anti-node* $n = 1$ in the figure, and is variously called the fundamental or the "lowest note" or the lowest mode of the string, the latter terms coming from the fact that the frequency is lowest for these standing waves. The higher n values correspond to harmonics of the fundamental, the overtones. Mathematically, it is not hard to check that

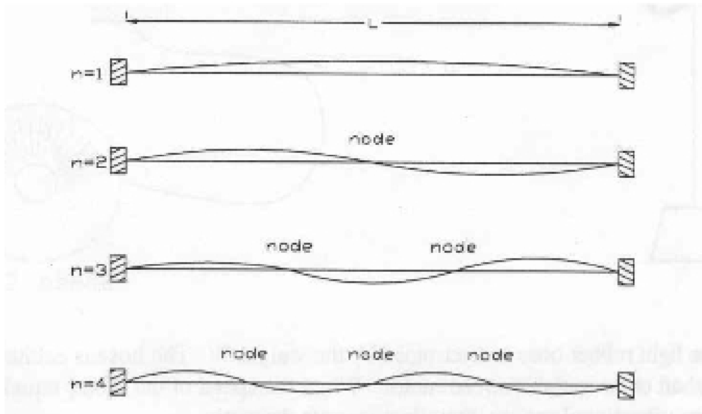

Figure 12: Standing waves on a string. n=1 corresponds to the fundamental mode; n>1 are harmonics.

$$\lambda_n = \frac{2L}{n}. \tag{19}$$

The frequencies are given by

$$f_n = \frac{v}{\lambda_n} = n f_1. \tag{20}$$

We should note that the string does not have to vibrate so that it has only one of these basic patterns of standing waves. In general, the excitation of a string (like that of a stringed instrument) will produce a mixture of these different patterns with different amplitudes simultaneously. The mix of overtones, among other features, distinguishes a violin from a piano or other instruments all playing the same "note", and lends music a richness that is hard to capture by our simple analysis. Still, understanding of the pure modes allows us to analyze even highly complicated mixtures of modes.

9.2 Equipment

In this lab, we will use a post attached to a wave driver (essentially a modified audio speaker) to stimulate the string at various frequencies. The wave driver is electrically driven by a function generator. Below are instructions for properly setting up the function generator.

Function generator

1. Push the on-off button (the green button on the left side of the front panel). The panel display should turn on.

2. Once the unit is on, press the "Output" button. It should glow, indicating that the source is putting out a voltage.

3. Press the button for a sine wave.

4. Press the gray button under the panel display "Ampl" to set the driver amplitude. Use the **arrow keys** on the right (just above the "Output" connector) to choose a digit and the **knob** to change the value. Increase the amplitude to 10.00 V peak to peak (V_{pp}). (In all cases, you can type the required number on the keypad rather than dialing it in).

5. Press the gray button under the panel display "Freq" to set the driver frequency. Again using the arrow keys and knob, set the frequency to 1.000 Hz.

   As you adjust the frequency of the driver, you should be able to hear the vibrations in the 50 Hz to about 15,000 Hz (15 kHz) range.

6. You are now ready to begin the experiment. Slowly increase the frequency to find the wave frequency that produces standing waves on your string.

9.3 Procedure

9.3.1 Determine μ and set up the string

1. *Determine μ*

   Cut a piece of string at least 4 meters in length and measure its length as precisely as you can. Then measure the mass of the string using the high-precision scale.

2. *Cut the string*

   Cut the string to a length somewhat longer than the workbench, tie one end to a post at one side of the workbench, pass the other end of the string over a pulley at the opposite side of the bench and attach a hanger to it. Adjust the string so that it passes through the slot in the post of the wave driver (described below).

   Do not tie the string to any part of the wave driver! The driver should only make gentle contact with the string.

   Q: Now that we have cut the string and tied some knots at the ends, how is it that the earlier measurements of the string's mass and length are still relevant?

3. *Choose and set the tension*

   Next, choose a tension corresponding to a hanging mass between 200 and 800 g. Make sure that the masses are placed securely on the hanger. Measure the total mass at the end of the string and note it down. Remember that the hanger itself has mass.

9.3.2 Qualitative observations of resonance

Start the function generator at the lowest frequency of the 10-Hz range and slowly turn the knob to increase the frequency. As you adjust the frequency, you will notice that, from time to time, the vibrations of the string suddenly grow in amplitude to reach a maximum when the string takes on one of the shapes depicted in the figure above, and then, as you increase the frequency further, the amplitude dies away just as suddenly. This is the phenomenon of resonance. When the driving frequency coincides with one of the natural frequencies of the medium (the string), the response of the medium increases sharply. Because this phenomenon is of central importance in many parts of physics and chemistry, spend a few moments just observing it qualitatively. If you go on and take an intermediate mechanics course, you will learn the mathematical theory behind resonance. Now, slowly raise and lower the driving frequency and observe as many standing wave resonances as you can. While you are at it, note how sharp each resonance is. That is, is the build-up slow or sudden as you gently ramp up or down the frequency? If it seems more or less to have nearly the same maximum amplitude over a wide range of frequencies, it would be hard to measure the resonant frequency with high precision. Conversely, if the maximum amplitude occurs discernibly over a narrow range of frequencies, you can measure the resonant frequency rather precisely. Pay attention to those features as you do the lab.

9.3.3 Quantitative study of wave characteristics

1. *Adjust to $n = 1$ and determine f_1*

   Carefully adjust the drive frequency so that the string is vibrating in the fundamental mode. Note the frequency on the function generator. Repeat the frequency measurement at least twice, each time moving away from resonance and moving back. Share the task of judging when you have hit resonance equally with your lab partner(s). Use the range of frequencies as a measure of experimental uncertainty.

2. *Measure $\lambda \pm \Delta\lambda$*

   Measure the distance between successive nodes. This will allow you to calculate the wavelength. Estimate how well you can locate the nodes, and hence the experimental uncertainty in the wavelength.[3]

3. *Repeat for harmonics*

   Measure as many harmonics as you can for this tension. You should be able to measure to at least $n = 5$.

4. *Repeat for two other tensions*

   Do the same things for two other tensions significantly different from the first. Carry out the analysis as described in the first section.

9.4 Analysis

You will find the velocity of the waves on the string graphically. If we rearrange Eq. 17, we can find an expression that treats the λ and f as two independent variables:

$$\begin{aligned} \lambda &= \frac{v}{f} \\ \lambda &= v\,T \end{aligned} \tag{21}$$

[3]For $n = 1$, the location of only one node is well defined. The other end of the string is being driven and thus has no node. Notice, however, that as you move along the string in the direction of the driver, the amplitude of the vibration is decreasing. You can still estimate the wavelength for the fundamental mode by extrapolating to the point where the the node *would* exist *if* the string continued without interruption. This point will often be very near the rod which the string is tied to, but its exact position is often difficult to pinpoint.

The new expression now predicts that a plot of λ vs. the period T will be a straight line with slope equal to v.

1. Enter your data into an Excel spreadsheet. Make sure to properly label each column including *units*.
2. Generate a plot of λ vs. T.
3. Use the regression analysis tool to find the best value and *uncertainty* of the slope.
4. Compare your results from the regression analysis to the predicted value of the velocity.

A Using the PASCO Motion Sensor & computer system

Several labs make use of the motion sensor and computer. This appendix outlines the general instructions for use of this system.

A.1 Setting up the motion sensor and computer system

If the computer is not turned on by the beginning of lab, please do so. Log on to the computer using your campus username and password. Connect the motion sensor to the PASPort USB Link, and then connect the USB Link to a USB port of the computer. A window should pop up with the title `PASPortal`. Select "Launch DataStudio."

A.2 Configuring the Motion Sensor

To configure the sampling rate of the motion sensor, start by clicking the `Setup` button. The Experiment Setup window will open. You can select what kind of data will be displayed by clicking the boxes next to `Position`, `Velocity` and `Acceleration`. Choose the sample rate by clicking the "+" or "-" buttons. If you are using more than one motion sensor, you can change the settings for each sensor individually by clicking on the stick-figure icon.

To choose between manual or automatic sampling, select the `Options...` button. A "Sampling Options" window will open. Click the `Automatic Stop` tab. If you wish to manually stop the data sampling, make sure the radio button `None` is selected. If you wish to sample data for a fixed amount of time, select `Time` and then enter the duration of the sample in seconds.

A.3 Taking a data sample

To take a data sample, simply click `Start`. If you are using the manual sampling mode, the data sampling will continue until you click `Stop`; otherwise, the sampling will stop at the end of the set time interval. The data are recorded in "Runs." You can take another sample by clicking `Start` again, a new sample will be recorded. The runs are numbered sequentially.

A.4 Adjusting the display

Viewing single/multiple Runs
You can choose which data to display on the graph area by selecting Runs from the `Data` drop-down menu in the `Graph` window. This allows you to view specific Runs without having to delete any data.

Selecting a Run
If you have multiple runs displayed, you can choose which is the currently selected Run by clicking on the labels in the graph legend.

Selecting a set of points
You will often need to analyze a subset of data. Simply drag a box around the points you want to analyze. To deselect highlighted data, click anywhere on the graph.

Adjusting the scale
To optimize the scale of your currently selected data, click the `Scale to fit` button in the upper-left corner of the `Graph` window. Alternatively, you can adjust the scale of each axis separately by clicking on one of the numbers along an axis and dragging the cursor. To pan the graph area, click on any axis itself and drag the cursor.

Displaying position, velocity, and acceleration
If the motion sensor is configured to record velocity and acceleration in addition to position, you can change the displayed measurement by clicking on the axis label and choosing among position, velocity, acceleration, and time. You can also create a new graph by double-clicking on `Graph` in the lower-left column and choosing the type to display.

Additionally, you can display all three of position, velocity, and acceleration versus a common time axis. To do this, click on the `Position/Velocity/Acceleration` labels from the `Data` column (where the Runs are listed) and drag-and-drop onto the graph area.

A.5 Exporting Data

To export data select `File` > `Export Data...` from the menu. A window will open with a list of runs. Select the run you wish to export and click `OK`. A file manager window will open by default in your "My Documents" fold. Type the name of the file in the "filename" box and then click `Save`. The data will be saved with a "txt" extension.

To export an image of a graph, select `Display` > `Export Picture...` from the top menu bar. This will save an image of the currently selected graph as a .bmp file to the specified location.

A.6 Opening Data in MS Excel

Start Excel. From the menu select `File` > `Open...` From the file manager window select the file that you exported from DataStudio. If you do not see the file listed go to "Files of types" and select either `All Files` or `Text Files`. Select the data file and click `Open`. The Text Import Wizard will open. Select the `Delimited` radio button, then click `Finish`.

B Keeping a lab notebook

Keeping a good lab notebook seems like a simple and obvious task, but it requires more care and thought than most people realize. It is a skill that requires consistent effort and discipline and is worth the effort to develop. Your lab notebook is your written record of everything you did in the lab. Hence it includes not only your tables of data, but notes on your procedure, and your data analysis as well. With practice, you will become adept at sharing your time fairly between conducting the experiment and recording relevant information in your notebook as you go along.

You want all this information in one place for three main reasons, and these reasons continue to be valid even after you leave the introductory physics laboratory. First, your lab notebook contains the information you will need to write a convincing report on your work, whether that report is for a grade in a course or a journal article. Second, you may need to return to your work months or even years after you have finished an experiment. It is surprising how often some early experiment or calculation is important in your later work. Hence, you need a reasonably complete account of what you have done. Third, your notebook is the source to which you turn in case someone questions the validity of your results. You should write as much detail as needed for someone, with only the lab manual and your lab notebook, to reproduce the experiments you performed and the calculations you did exactly.

Your notebook therefore serves two purposes that may not be completely compatible with each other. On the one hand, you should write things down pretty much as they occur and before you have a chance to forget them, so that you have a complete record of your work in the lab. On the other hand, your notebook should be reasonably neat and well-organized, partially so you can find things and partially so that if anyone questions your results, not only will they be able to find things, but the layout of your notebook will suggest that you investigated the problem carefully and systematically.

You should use a bound lab notebook (that is, not a looseleaf notebook). So-called quadrille notebooks (with rectangular grids on each page) are particularly handy for making graphs and tables. If you wish to add a graph done on a computer to the notebook or a graph done on regular graph paper, you may simply tape or glue the graph into your notebook.

Next we will discuss some of the information that goes into your lab notebook.

B.1 Title, date, equipment

You should begin each new experiment on a fresh page in your notebook. Start with the date and a brief title for the experiment—just enough to remind you what that section of your notebook is about. Identify large pieces of equipment with manufacturer's name, the model, and the serial number. With this information, you can repeat the experiment with the identical equipment if for some reason you are interrupted and have to return to the equipment much later. Or, if you are suspicious of some piece of equipment, having this information will let you avoid that particular item.

B.2 Procedure

You won't be expected to copy the procedure printed in your lab manual into your notebook, but be sure to indicate where the procedure can be found. It is essential that you note any deviation from the procedure described in the manual. Write in complete sentences and complete paragraphs. This is part of the discipline required for keeping a good lab notebook. Single words or phrases rapidly become mysterious, and only with a sentence or two about what you're measuring, such as the period of the pendulum as a function of length, will you be able to understand later on what you did. Give more details where necessary, if for example the lab manual does not give a more detailed procedure or if you depart from the procedure in the manual.

B.3 Numerical Data

When recording numerical data, keep your results in an orderly table. You should label the columns, and indicate the units in which quantities are measured. You should also indicate the uncertainty to be associated with each measurement. If the uncertainty is the same for all data of a certain set, you can simply indicate that uncertainty at the top of the column of those data.

You will need at least two columns, one for the independent variable and one for each dependent variable. It's also good to have an additional column, usually at the right-hand edge of the page, labeled "Remarks." That way, if you make a measurement and decide that you didn't quite carry out your procedure correctly, you can make a note to that effect in the "Remarks" column. (For example, suppose that you realize in looking at your pendulum data that one of your measurements must have timed only nine swings instead of ten. If you indicate that with, say "9 swings?" you could justify to a suspicious reader your decision to omit that point from your analysis.)

B.4 Do not erase

Write only in ink. Pencil is too easy to erase and is smudged over time. Never erase anything; original data, calculation and so on, may turn out to be correct after all, and in any case, you want to keep a complete record of your work, even the false starts. If you believe that a calculation, for example, is wrong, it is better to cross it out and make a note in the margin.

B.5 Sequences of Measurements

You will often be performing experiments in which you have two independent variables. Usually in such experiments you fix the value of one independent variable and make a series of measurements working through several values of the other variable. Then you change the value of the first variable and run through the measurements with the other variable again; then you change the first independent variable again, make another set of measurements, and so on. It's usually easier to set up this sort of sequence in your notebook as a series of two-column tables (or three columns with "Remarks") rather than a big rectangular grid. "Title" each table with the value of the independent variable that you're holding fixed, and keep the format of all the tables the same.

B.6 Comment on Results

Once you have completed the experiment and performed any necessary calculations in the notebook, you should look back to the main goal and write down to what extent it was achieved. If, for example, you were making a measurement of g, you should include a clear statement of the value of g along with its uncertainty. Be aware that there are often secondary goals as well (to become familiar with a particular physical system or measurement technique, for example). Comment on your success in attaining these goals as appropriate. This serves as a statement of conclusion and gives you the chance to make sure the lab was completed thoroughly and to your satisfaction.

B.7 Guidelines to keeping a good notebook

1. Bound Notebook: No spiral bound, loose leaf or perforated page notebook. Lab notebooks are a *permanent record* of the work done in lab. The integrity of the notebook should not be comprised by tearing out pages.

2. Keep a record. Write down names, title, time, places and dates.

3. Be generous with use of pages. Start each experiment on a fresh page. leave some blank pages between experiments in case you need to add tables or graphs.

4. Do not erase, redact or scribble out possible mistakes. Draw one single line through any value or calculation you suspect may be wrong.

5. Define terminology and variables with units.

6. Sketch the experimental setup. Label the relevant parts and indicated measured quantities. List equipment used.

7. Be complete. Every experiment has a introduction (pre-lab note), procedure, analysis, results and summary. Make sure all tables, graphs, diagrams and calculations are in your notebook for reference.

8. Annotate! Each section of experiment should begin with a paragraph discussing what the section is about. Do not leave it to a reader to guess.

9. Be organized, neat and legible. The reader should not to have struggle to decipher your notes. Don't be cryptic.

10. Answer all questions and exercises. Use your answers to inform your discussion of your formal report or exit interview. Always answer questions and exercise from a physics point of view.

C Graphical Presentation of Data

C.1 Introduction

"Draw a picture!" is an important general principle in explaining things. It's important because most people think visually, processing visual information much more quickly than information in other forms. Graphing your data shows relationships much more clearly and quickly, both to you and your reader, than presenting the same information in a table.

Typically you use two levels of graphing in the lab. A graph that appears in your final report is a "higher-level" graph. Such a graph is done neatly (and almost always with a graphing program), following all the presentation guidelines listed below. It's made primarily for the benefit of the person reading your report. "Lower-level" graphs are rough graphs that you make for your own benefit in the lab room; they're the ones the lab assistants will hound you to construct. These lower-level graphs tell you when you need to take more data or check a data point. They're most useful when you make them in time to act on them, which means that you should get in the habit of graphing your data in the lab while you still have access to the equipment. (That's one reason, in fact, that we recommend that you leave every other sheet in your lab notebook free, so you can use that blank sheet to graph your data.) In graphing your data in the lab, you don't need to be too fussy about taking up the whole page or making the divisions nice. You should label the axes and title the graph, though. Since we do much of our data analysis in Excel, be sure to print out relevant graphs and analyses and tape them into your notebook.

Flaky data points show up almost immediately in a graph, which is one reason to graph your data in the lab. Skipping this low-level graphing step can allow problems in the data collection to propagate undetected and require you to perform the experiment again from the beginning. Graphing each point as you take it is probably not the best idea, though. Doing so can be inefficient and may prejudice you about the value of the next data point. So your best bet is to take five or six data points and graph them all at once.

Graphing your data right away also flags regions in your data range where you should take more data. Typically people take approximately evenly-spaced data points over the entire range of the independent variable, which is certainly a good way to start. A graph of that "survey" data will tell you if there are regions where you should look more closely; regions where you graph is changing rapidly, going through a minimum or maximum, or changing curvature, for example. The graph helps you identify interesting sections where you should get more data, and saves you from taking lots of data in regions where nothing much is happening.

C.2 Analyzing your Graph

"Graphical data analysis" is usually a euphemism for "find the slope and intercept of a line." You will find this semester that you spend a lot of time redrawing curves by employing the "method of straight line graphing" so that they turn into straight lines, for which you can calculate a slope and an intercept. This process is so important that, although we have a fond hope that you learned how to do this in high school, we're going to review it anyway.

Presumably you have in front of you some graphed data that look pretty linear. Start by drawing in by eye the line that you think best represents the trend in your data. An analytical procedure exists to draw such a line, but in fact your eyeballed line will be pretty close to this analytically-determined "best" line. Your job now is to find the slope and intercept of that "best" line you've drawn.

Next we tackle the question of finding the slope and intercept of that line. As usual, we will assume that the line is described by the equation

$$y = mx + b \tag{22}$$

where m is the slope of the line and b is the y-intercept.

Two points determine a line, and a line is also described completely by its slope and intercept. (This should make a certain amount of sense. You put in two pieces of information, you get out two pieces of information.) Your first task is therefore to choose two points on your line. These two points describe the line, so they need not (and most likely *will* not) be data points. They should be far apart on the graph, to minimize the effects of the inevitable experimental uncertainty in reading their locations from the graph paper. The two points should also be located at easy-to-read crossings on the graph paper. Mark each of those points with a heavy (but not too large) dot and draw a circle around the dot. Read the coordinates of each point off the graph.

The slope of the line is defined as the change in y (the vertical coordinate) divided by the corresponding change in x (the horizontal coordinate). (You may know this in some other form, such as "rise over run.") To calculate the slope, use

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}, \tag{23}$$

substituting your values for the points (x_1, y_1) and (x_2, y_2). For example, if your points are $(1.0\ \text{s}, 8.8\ \text{m/s})$ and $(6.0\ \text{s}, 46.3\ \text{m/s})$, then $m = 7.5\ \text{m/s}^2$. (Notice that the units of m are the units of "rise-over-run.") Now that you have the slope, find the intercept from

$$\text{intercept} = b = y_1 - mx_1. \tag{24}$$

That is, you can read b directly off off the graph, or you can use the slope and one point to determine b. Use either point for (x_1, y_1). Both lie on the line, so either will work. In the example above, we get that $b = 1.3$ m/s. (Notice that the units of b are those of the y variable.)

Once you have determined the values of m and b from the graph, you can quote the equation for your straight line. For example, if $m = 7.5\ \text{m/s}^2$ and $b = 1.3$ m/s, then the equation of your straight line is

$$y = (7.5\ \text{m/s}^2)\text{x} + 1.3\ \text{m/s}. \tag{25}$$

This equation gives a **complete** description of the line and the job is done.

C.3 Uncertainty Bars

Individual data points plotted on any graph should usually include uncertainty bars (sometimes misleadingly called "error bars") showing the uncertainty range associated with each data point. You should show both vertical and horizontal uncertainty bars, if the uncertainties are large enough to be visible on the graph. (If they aren't large enough, you should mention this in your report so we don't think you've forgotten them.) You can draw uncertainty bars by indicating the "best guess" value (typically the measured value or average of several measurements) with a dot, and drawing an "I-bar" through the dot with its length indicating the range in the uncertainty. When you use Excel or the Graphical Analysis data analysis package, this step can be done for you — with severe limitations. Such a package will typically only determine error bars by considering the scatter of the individual data points about the best-fit straight line. While this is helpful in providing a consistency check for the data, it does not tell the complete story of the uncertainties in your data. That is, unless you use a more advanced feature of such an analysis program, it has no way of knowing about the uncertainties that were inherent in your measured values because of the measurement apparatus. Only you can decide how accurately you used the meter stick, or how quickly you were able to react when starting and stopping a stopwatch. You will not always be expected to put error bars on all of your plotted points, but you should know how it is done and be able to apply it to the first lab.

C.4 Graphical Presentation Guidelines

Use these guidelines for "higher-level" graphs.

1. Use a computer to produce your graphs. Hand-plotted graphs are fine as long as they comply with the following guidelines.

2. Scale your axes to take the best possible advantage of the graph paper. That is, draw as large a graph as possible, but the divisions of the graph paper should correspond to some nice interval like 1, 2, or 5 (times some power of 10). If you have to make the graph smaller to get a nice interval, make it smaller, but check that you've picked the nice interval that gives you the largest graph. Making the graph large will display your data in as much detail as possible. When using log-log or semi-log paper, choose paper with the number of cycles that gives the largest possible graph.

3. The lower left-hand corner need not be the point $(0, 0)$. Choose the range of values for each axis to be just wide enough to display all the data you want.

4. Mark the scale of each axis (the number of units corresponding to each division) for the entire length of the axis.

5. Label both axes, identifying the quantity being plotted on each axis *and* the units being used.

6. Give each graph a title or provide a figure caption. The title should summarize the information contained in the axes and also gives any additional information needed to distinguish this graph from other graphs in the report.

7. Give each graph a number (e.g., "Figure 2"), which you can use in the body of the report to refer quickly to the graph.

8. If you calculate the slope and intercept of the graph from two points (rather than using linear regression), indicate the two points you used on the graph. Draw the line through the two points, label it "Best-fit line" (or something similar), and give its slope and intercept on the graph in some large clear space.

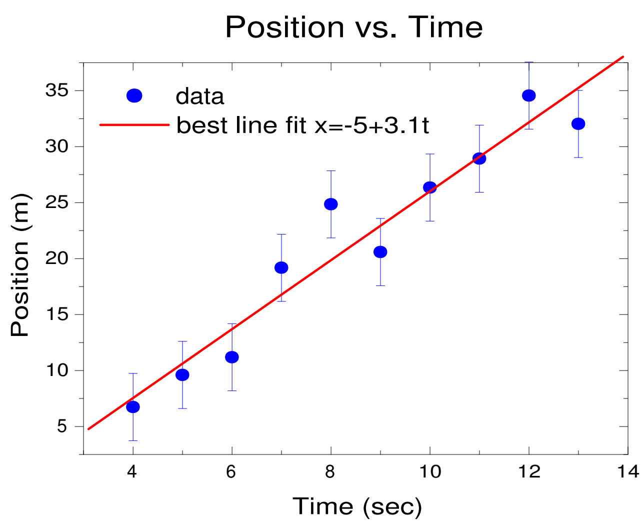

Figure 13: Position vs. Time
This is a sample graph illustrating all the features of a "high-level" graph. The solid line represents the best fit to the sample data.

C.4.1 Graphing Checklist

- Axes scaled correctly with divisions equal to "nice" intervals (1, 2, 5 or 10);
- Graph drawn to as large a scale as possible;
- Scales on axes labeled for entire length;
- Axes labeled, including units;
- Graph titled and numbered; and
- Points used to calculate slope and intercept clearly marked, if that method is used.

D Guidelines for Formal Laboratory Reports

The formal lab report is a paper, less than or equal to six pages in length, presenting your work on the experiment. It should be written for someone who has a physics background equivalent to Physics 116, *but who does not know anything about the experiment and the measurements you carried out*. There are three principal components to every formal report:

- *Format*: the organization and presentation of the report.
- *Composition*: the style in which the report is written.
- *Content*: the subject matter of the report.

All three are essential for writing a complete and self-consistent report. The purpose of the reports is to test both your analytical skills and your writing skills at communicating physical concepts.

If we find that the report could be substantially improved by rewriting, we may ask for a revised report before a grade is given for the lab.

D.1 Format

The report should be formatted in a way that clearly presents all the relevant information to the reader: text, equations, figures, etc. It would help us (and you) if you adopted the following format:

- Typeset using a word-processor.
- Lines double spaced.
- 12 point Times New Roman font.
- 1" left and right margins.
- No more than 6 pages.

You will probably want to include other forms of information, such as equations, diagrams, tables, and so forth. These have their own formatting conventions.

D.1.1 Equations

Equations should be centered on separate lines from the text of the report. Each equation should be numbered, preferably along the right margin, for easy reference. You should make sure that every variable is defined in an equation; for instance, consider Einstein's famous equation

$$E = mc^2, \tag{26}$$

where E is energy, m is mass, and c is the speed of light. Note that this equation (Eq. 26) could also have appeared directly in the text as $E = mc^2$, although the latter format makes it difficult to refer to it later in the document. In either case, punctuate equations as you would any other text.

Equations are especially useful when stating the theoretical background of the report. Reference equations that are used in your derivations and give final results — try not to include the derivations themselves. (Keep derivations in your notebooks.)

D.1.2 Figures

Diagrams of experimental apparatus should be simple yet illustrative of the setup. The relevant parts should be labeled and the relevant measured quantities indicated. Each diagram should have a figure number (e.g., Figure 1:) and caption below the diagram. The caption should be a concise description of the figure and any important parts. Please generate your own diagrams — do not download figures from the internet.

D.1.3 Graphs

The title of a graph should clearly indicate which two quantities are plotted. The title convention for a graph is Y (vertical axis) vs. X (horizontal axis). The axes should be labeled and include units. Graphs use the same convention of numbering and captions as diagrams (Figure 1:). The caption should be a brief description of the graph and the quantities plotted. Adjust scale of axes so data points fill the whole graph. Empty space is a waste. For more details refer to Appendix C, Graphical Presentation of Data.

D.1.4 Tables

Tables are generally only useful only when the relevant data are not included in a graph; otherwise they should be recorded as a table in your notebook or possibly electronically in a spreadsheet (for especially large tables that are generated by devices such as the motion sensor). Data tables should be organized in columns. The head of each column should be labeled and include units. If the quantities in the column all have the same uncertainty, then the uncertainty can be indicated at the head of the column as well; for example, Time ($\pm$0.001 sec). Each table should have a descriptive title, starting with a number (Table 1:) for easy reference. Do not split tables across pages of the report.

D.2 Organization

While we leave to you the exact organizational details, you will likely wish to include the following:

1. **Title:** The title should be a simple descriptive phrase, centered at the top of the first page of the report. Also include your name, the date, the lab section and the name(s) of your partner(s).

2. **Introduction:** The introduction is a short, single paragraph statement of the experiment. What is the purpose, the *main* goal, of experiment and why is the experiment a worthwhile means of exploring a particular physical concept?

3. **Theoretical background:** The theoretical background should state what the underlying physics of the experiment is. What the theory predicts, what assumptions have been made, and how the experiment relates to the theory of the physics being studied. Terminology specific to the experiment should be defined. Often, the theory can be best expressed analytically in the form of an equation. Define the quantities to be determined and how they are related to the directly measured quantities.

4. **Experimental technique:** The experimental technique should be a detailed *narrative* of the experimental procedure. *What* was measured and *how* was it measured? Include a simple diagram of the apparatus whenever possible. Indicate the primary sources of measurement uncertainty. Give numerical estimates of uncertainties associated with each directly measured quantity.

5. **Data, analysis, and results:** Display the data in one or more appropriate forms (tables, graphs, etc). Discuss how the final results are obtained. Give estimates of the uncertainty of the results based upon measurements uncertainties. Be sure to include some discussion of experimental uncertainties and how those uncertainties affect the evaluation of your results.

6. **Discussion of results and Conclusion:** The conclusion should reflect your overall understanding of the experiment, i.e., what have you learned about the particular subject of physics studied in the experiment? It should consist of a logical sequence of statements substantiated by the evidence presented in the report. Was the goal of the experiment accomplished? Were the experimental results *consistent* with theoretical expectations? That is, do they agree within the range of uncertainty? What are the implications of your results? It is good practice to restate any numerical results in the conclusion for easy review by the reader.

D.3 Composition

You should think of a formal lab report as a paper, similar to papers you have written in other courses. Scientific writing has its own idiom, however, and includes the following conventions:

- **Write in narrative prose, not outline form.** This is especially true when writing sections of the report like the experimental technique. Do not recite the procedure outlined in the lab manual.

- **Use simple, complete sentences.** Sentences are expressions of one complete thought, fact or idea. The simpler the sentence, the better. Avoid excessive use of qualifiers, modifier and subordinate clauses and phrases. Get to the point and stay on topic.

- **Write in paragraph style.** The paragraph is the building block of the report. Each paragraph should address one topic of the report. (Please indent properly!)

- **Use correct terminology, spelling and grammar.** There are some words in everyday language that have specific meaning in the context of physics. Make sure the terminology is consistent with the subject matter of the report. Always use the standard spelling of words.

- **Write formally; do not use slang or colloquialisms.** Do not write in a casual manner; for example, the word "plug", as in, "I plugged the numbers into the equation.." This is a sloppy, lazy style of writing. Formal reports should be written in a formal style.

D.4 Content

The content of a lab report can be divided into two categories:

- Technical content which includes text, figures, equations and tables.
- Topical content which is the discussion of the subject the report is about.

Both are necessary for writing a complete report. The specific content will vary from experiment to experiment. The topical content is probably the most challenging part of a report to write. A report that is *technically* correct, i.e., is formatted properly, is free of spelling and grammatical errors and contains all the appropriate figures, equations, and tables, etc., is still incomplete if the *topical* content is lacking. In general there are five things that should be considered when writing a report:

- **Clarity**: the underlying principles are clearly articulated, all relevant terminology is defined. You should be specific in the language used. Avoid vague or ambiguous statements.
- **Completeness**: all elements of the report are present. Missing or omitted content will mislead or confuse the reader.
- **Conciseness**: specific and to the point. The writer should avoid redundant, irrelevant or circuitous statements. Stay on topic.
- **Consistency**: all elements of the report direct the reader to a single, logical conclusion. Avoid illogical, erroneous, unsubstantiated, specious, irrelevant statements and contradictions.
- **Continuity**: all elements of the report follow a logical order. The discussion is constructed in a sequential manner. Avoid incoherent, disorganized statements.

D.5 Questions and Exercises

In some experiments specific questions and exercises will be asked. The purpose of the questions and exercises is to motivate the discussion. Questions and exercises should be answered within the body of the report and always from a *physics* point of view. The report is incomplete without answers to questions and exercises.

E Experimental Uncertainty Analysis

An intrinsic feature of every measurement is the uncertainty associated with the result of that measurement. No measurement is ever exact. Being able to determine easily and assess intelligently measurement uncertainties is an important skill in any type of scientific work. The measurement (or experimental) uncertainty should be considered an **essential** part of every measurement.

Why make such a fuss over measurement uncertainties? Indeed, in many cases the uncertainties are so small that, for some purposes, we needn't worry about them. On the other hand, there are many situations in which small changes might be very significant. A clear statement of measurement uncertainties helps us assess deviations from expected results. For example, suppose two scientists report measurements of the speed of light (in vacuum). Scientist Curie reports 2.99×10^8 m/sec. Scientist Wu reports 2.98×10^8 m/sec. There are several possible conclusions we could draw from these reported results:

1. These scientists have discovered that the speed of light is not a universal constant.
2. Curie's result is better because it agrees with the "accepted" value for the speed of light.
3. Wu's result is worse because it disagrees with the accepted value for the speed of light.
4. Wu made a mistake in measuring the speed of light.

However, without knowing the uncertainties, which should accompany the results of the measurement, we cannot assess the results at all!

E.1 Expressing experimental uncertainties

Suppose that we have measured the distance between two points on a piece of paper. There are two common ways of expressing the uncertainty associated with that measurement: absolute uncertainty and relative uncertainty. In both ways the measured quantity is expressed in the form:

$$x_{measured} = x_{best} \pm \Delta x \qquad (27)$$

Here x_{best} is the best measured value, usually from an average of a set of measurements, and Δx is the uncertainty in the best measured value. The measurement is always a *range* of values, not just the best value.

E.1.1 Absolute uncertainty

We might express the result of the measurement as

$$5.1 \text{ cm} \pm 0.1 \text{ cm} \qquad (28)$$

By this we mean that the result (usually an *average* result) of the set of measurements is 5.1 cm, but given the conditions under which the measurements were made, the fuzziness of the points, and the limitations of our distance measuring equipment, it is our best judgment that the "actual" distance might lie between 5.0 cm and 5.2 cm.

E.1.2 Relative (or percent) uncertainty

The relative uncertainty is defined:

$$f_x = \frac{\Delta x}{|x_{best}|} \tag{29}$$

We might express the same measurement result as

$$x_{measured} = x_{best} \pm f_x \tag{30}$$

For example:

$$5.1 \text{ cm} \pm 2\%.$$

Here the uncertainty is expressed as a percent of the measured value. Both means of expressing uncertainties are in common use and, of course, express the same uncertainty.

E.1.3 Graphical Presentation: Uncertainty Bars

If we are presenting our data on a graph, it is traditional to add uncertainty bars (also mistakenly but commonly called "error bars") to the plotted point to indicate the uncertainty associated with that point. We do this by adding an "I-bar" to the graph, with the I-bar extending above and below

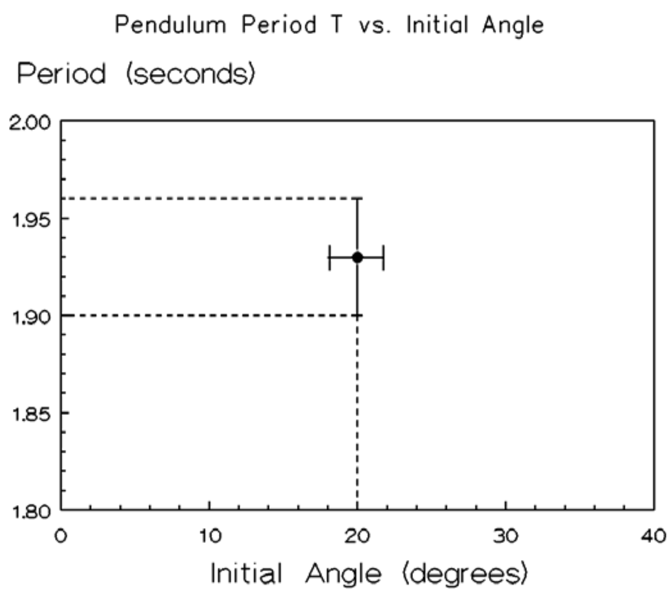

Figure 14: Plotted data points with uncertainty bars.
The point has a y uncertainty of ± 0.3 and an x uncertainty of ± 2.

the plotted point (if the numerical axis is vertical). (If we are plotting a point on an (x, y) graph and there is some uncertainty in both the x and y values, then we use a horizontal I-bar for the x uncertainty and a vertical I-bar for the y uncertainty.

E.1.4 An aside on significant figures

The number of significant figures quoted for a given result should be consistent with the uncertainty in the measurement. In the previous example, it would be inappropriate to quote the results as 5 cm ± 0.2 cm (too few significant figures in the result) or as 5.132 cm ± 0.2 cm (too many significant figures in the result). The uncertainties themselves, since they are estimates, are usually quoted with only one significant figure, or in some cases, (for very high precision measurements, for example) with two significant figures.

E.1.5 Rules for Significant figures

1. All nonzero digits are significant. Ex.: 1-9.
2. Zeros between nonzero digits are significant. Ex.: 230504 (6 significant figures).
3. Leading zeros to left of nonzero digit are not significant. Such zeros only indicate position of decimal point. Ex.: 0.002 (1 significant digit).
4. Trailing zeros to right of decimal point are significant. Ex.: 0.0340 (3 significant digits).
5. Trailing zeros to the left of the decimal point may or may not be significant. Ex.: 50,600 (3, 4 or 5 significant figures?) Avoid this usage by adding a decimal point (50,600. has 5 significant figures) or by using scientific notation (5.06×10^4, or equivalently 5.06E4, has 3 significant figures).
6. When adding or subtracting numbers, the final answer is round off to the decimal place equal to the number with the fewest decimals.
7. When multiplying or dividing numbers, the final answer is rounded to the same number of significant figures as the number with the fewest significant figures.

E.2 Systematic errors, precision and random effects

Why aren't measurements perfect? The causes of measurement uncertainties can be divided into three broad categories: *systematic problems*, *limited precision*, and *random effects.*

E.2.1 Systematic errors

Systematic errors occur when a piece of equipment is improperly constructed, calibrated, or used. For example, suppose the stopwatch that you are using runs properly at 20° C, but you happen to be using it where the temperature is closer to 30° C, which (unknown to you) causes it to run 10% too fast.

If you know that a systematic problem exists, you should fix the problem (for example, by calibrating the stopwatch). The most appropriate thing to do with systematic problems in an experiment is to find them and to eliminate them. Unfortunately, no well-defined procedures exist for finding systematic errors: the best you can do is to be clever in anticipating problems and alert to trends in your data that suggest their presence. In some cases, you are aware of systematic effects but you are unable to determine their effects absolutely precisely. It is appropriate in such cases to include in the stated measurement uncertainty a contribution due to the imprecise knowledge of the systematic effects.

E.2.2 Limited precision

Limited precision is present because no measurement device can determine a value to infinite precision. Dials and linear scales (such as meter sticks, thermometers, gauges, speedometers, and the like) can *at best* be read to within one tenth (or so) of the smallest division on the scale. For example, the smallest divisions on a typical metric ruler are 1 mm apart: the *minimum* uncertainty for any measurement made with such a ruler is therefore about ±0.1 mm. This last statement is not an absolute criterion, but a rule-of-thumb based on experience.

For measuring devices with a digital readout (a digital stopwatch, a digital thermometer, a digital voltmeter, and so on), the minimum uncertainty is ±1 in the **last displayed digit**. For example, if your stopwatch reads 2.02 seconds, the "true" value for the time interval may range anywhere from 2.010...01 to 2.029999 seconds. (We don't know whether the electronics in the watch rounds up or down.) So, in this case, we must take the uncertainty to be at least ±0.01 second.

For both digital and "analog" devices, we need to be aware of what is called **calibration uncertainty**. For example, a manufacturer may state that a particular voltmeter has been calibrated to ±0.01 volt, that is, the readings of this meter agree with some standard, say, from the National Institute of Standards and Technology, to within 0.01 volt. Note that this uncertainty is different from the uncertainty in the displayed digits mentioned above. Although the manufacturer usually chooses the number of display digits so that the two types of uncertainty are about the same, that is not necessarily the case. In both of the cases described above, these rules of thumb are meant to represent the minimum possible uncertainties for a measured value. Other effects might conspire to make measurements more uncertain than these limits, but there is nothing that you can do to make the uncertainties smaller, short of buying a new device with a finer scale, or more digits, or a lower calibration uncertainty.

Accuracy and Precision. Calibration uncertainty is really an issue of what scientists call **accuracy**: how accurate is a particular measurement in terms of an accepted set of units such as volts, seconds, and so on? Accuracy is to be distinguished from **precision**, which is a question of the *internal* consistency and repeatability of a set of measurements without worrying about matching those measurements with other measurements or with standard units. For example, if I am using a stopwatch that is running too fast by a fixed amount, my set of timing measurements might be very *precise* if they are very repeatable. But they are not very *accurate* because my time units are not well matched to the standard second.

In both of the cases described above, these rules of thumb are meant to represent the *minimum* possible uncertainties for a measured value. Other effects might conspire to make measurements *more* uncertain than these limits, but there is nothing that you can do to make the uncertainties smaller, short of buying a new device with a finer scale or more digits or a lower calibration uncertainty.

E.2.3 Random effects

Random effects show up in the spread of results of repeated measurements of the same quantity. For example, five successive stopwatch measurements of the period of a pendulum might yield the results 2.02 s, 2.03 s, 2.01 s, 2.04 s, 2.02 s. Why are these results different? In this case, the dominant effect is that it is difficult for you to start and stop the stopwatch at *exactly* the right instant: no matter how hard you try to be exact, sometimes you will press the stopwatch button a bit too early and sometimes a bit too late. These unavoidable and essentially random measurement effects cause the results of successive measurements of the same quantity to vary.

In addition, the quantity being measured itself may vary. For example, as the temperature in the lab room increases and decreases, the length of the pendulum may increase and decrease. Hence, its period may change. In laboratory experiments, we try to control the environment as much as possible to minimize these fluctuations. However, if we are measuring the light radiated by a star, there is no way to control the star and its inherent fluctuations. Furthermore, fluctuations in the interstellar medium and in the earth's atmosphere may cause our readings to fluctuate in a random fashion.

In our analysis, we will assume that we are dealing with random effects, that is, we will assume that we have eliminated systematic errors. For most experiments we try to remove systematic errors and reduce calibration uncertainties and the effects of limited precision so that random effects are the dominant source of uncertainty.

E.3 Determining experimental uncertainties

There are several methods for determining experimental uncertainties. Here we mention three methods, which can be used easily in most of the laboratory measurements in this course.

E.3.1 Estimate Technique

In this method, we estimate the precision with which we can measure the quantity of interest, based on an examination of the measurement equipment (scales, balances, meters, etc.) being used and the quantity being measured (which may be "fuzzy," changing in time, etc.). For example, if we were using a scale with 0.1 cm marks to measure the distance between two points on a piece of paper, we might estimate the uncertainty in the measured distance to be about ± 0.05 cm, that is, we could easily estimate the distance to within 1/2 scale marking. Here we are estimating the uncertainty due to the limited precision or calibration uncertainty of our equipment.

E.3.2 Sensitivity Estimate

Some measurements are best described as comparison or "null" measurements, in which we balance one or more unknowns against a known quantity. For example, in some electrical circuit experiments, we determine an unknown resistance in terms of a known resistance by setting a certain potential difference in the circuit to zero. We can estimate the uncertainty in the resulting resistance by slightly varying the known resistance to see what range of resistance values leads to a "zero potential difference condition" within our ability to check for zero potential difference.

E.3.3 Repeated Measurement (Statistical) Technique

If a measurement is repeated in independent and unbiased ways, the results of the measurements will be slightly different each time. A statistical analysis of these results then, it is generally agreed, gives the "best" value of the measured quantity and the "best" estimate of the uncertainty to be associated with that result.

Mean Value (Average Value) The usual method of determining the best value for the result is to compute the "mean value" of the results: If x_1, x_2, ..., x_N are the N results of the measurement of the quantity x, then the mean value of x, usually denoted by $\overline{x}$, is defined as

$$\overline{x} = \frac{x_1 + x_2 + ... + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{31}$$

Standard Deviation The uncertainty in the result is usually expressed as the "**root-mean-squared deviation**" (also called the "**standard deviation**"), usually denoted as σ_x (Greek letter sigma). Formally, the standard deviation is computed as

$$\sigma_x = \sqrt{\frac{(x_1 - \overline{x})^2 + ... + (x_N - \overline{x})^2}{N-1}} \tag{32}$$

Let's decipher Eq. 32 in words. Eq. 32 tells us to take the difference between each of the measured values (x_1, x_2, ...) and the mean value $\overline{x}$. We then square each of the differences (so we count plus and minus differences equally). Next we find the average of the squared differences by adding them up and dividing by the number of measured values. (The -1 in the denominator of Eq. 32 is a mathematical refinement to reflect the fact that we have used the N values once to calculate the mean.) Finally, we take the square-root of that result to give a quantity which has the same units as the original x values. Figure 15 shows graphically the association of the standard deviation with the "spread" of x values in our set of measurements.

E.3.4 Interpretation

What meaning do we give to the uncertainty determined by one of the methods given above? The usual notion is that it gives us an estimate of the spread of values we would expect if we repeated the measurements many times (being careful to make the repetitions independent of one another). For example, if we repeated the distance measurements cited in the previous case study many times, we would expect most (statisticians worry a lot about how to make "most" more quantitative) of the measurements to fall within ± 0.06 cm of each other.

95% Confidence Range Sometimes uncertainties are expressed in terms of what is called the "95% Confidence Range" or "95% Confidence Limits." These phrases mean that if we repeat the measurements over and over many times, we expect 95% of the results to fall within the stated range. (It also means that we expect 5% of the results to fall outside this range!) Numerically, the 95% Confidence Range is about two times the standard deviation. Thus, we expect 95% of future measurements of that quantity to fall in the range centered on the mean value. Figure 15 shows the 95% confidence range for a particular distribution of values.

Asides: The standard deviation value is the "68% Confidence Range." The actual multiplicative factor for the 95% Confidence Range is 1.98 if the measurements are distributed according to the so-called "normal" ("Gaussian") distribution. But, for all practical purposes using 2 is fine for estimating the 95% Confidence Range.

In general, we cannot expect exact agreement among the various methods of determining experimental uncertainties. As a rule of thumb, we usually expect the different methods of determining the uncertainty to agree within a factor of two or three.

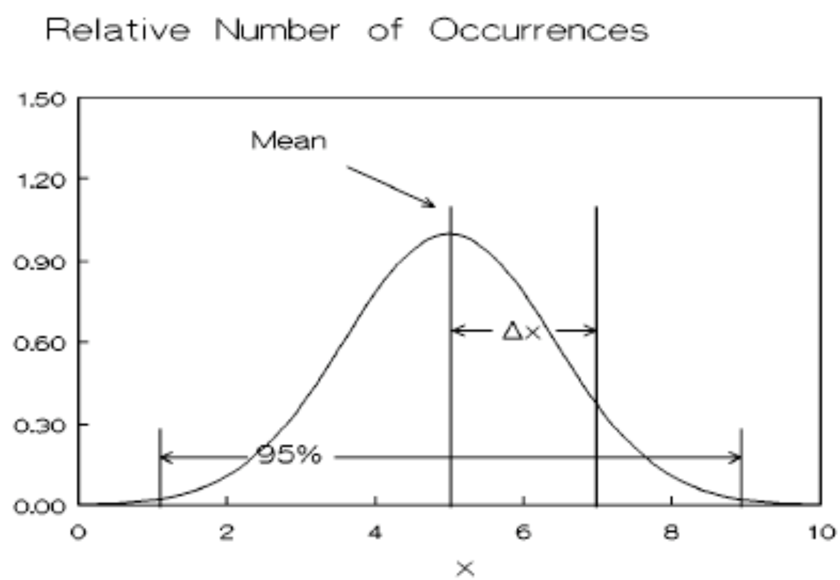

Figure 15: Distribution of x values from a set of measurements.
The mean value (average value) and standard deviation are indicated. The 95% Confidence Range is shown.

E.3.5 Example

Suppose that five independent observers measure the distance between two rather fuzzy marks on a piece of paper and obtain the following results:

$$
\begin{aligned}
d_1 &= 5.05 \text{ cm} \\
d_2 &= 5.10 \text{ cm} \\
d_3 &= 5.15 \text{ cm} \\
d_4 &= 5.20 \text{ cm} \\
d_5 &= 5.10 \text{ cm}
\end{aligned}
$$

If the observers were using a scale with 0.1 cm markings, the estimate technique would suggest an uncertainty estimate of about ±0.05 cm. The statistical technique yields a mean value $d = 5.12$ cm and for the standard deviation 0.057 cm ≈ 0.06 cm. We see that in this case we have reasonable agreement between the two methods of determining the uncertainties. We should quote the result of this measurement as 5.12 cm ± 0.06 cm or 5.12 cm ± 1%.

In practice, it is not really meaningful to use the statistical estimation procedure unless you have at least ten or so independent measurements. For small data sets, you should simply estimate the uncertainty using one of the other methods cited above.

Exercise 1 Using the data listed above, carry through the calculation of the mean, the standard deviation, and the 95% confidence range.

E.3.6 Standard Deviation of the Mean

A further question arises: suppose we repeat sets of measurements many times and for each set we compute a mean value. How close do we expect the mean values to lie with respect to each other? Perhaps surprisingly, we can estimate this result from the data in a single run. Although we will not provide an explicit justification here (see Taylor [1] or Bevington [2]), the answer is given by the so-called **standard deviation of the mean** :

$$\alpha = \frac{\sigma}{\sqrt{N}} \tag{33}$$

where N is the number of data points that go into the calculation of the mean and is the standard deviation for one "run." Note that we expect the *mean values* to be more closely clustered than the individual measurements.

In the scientific research literature, the standard deviation of the mean (or twice that for the 95% confidence range) is usually (but not always) the quantity cited for the uncertainty in the measured quantity.

Perhaps it is worthwhile emphasizing at this point that our analysis applies only to "random" uncertainties, that is, essentially uncontrollable fluctuations in equipment or in the system being measured, that collectively lead to scatter in our measured results. We have (implicitly) assumed that we have eliminated (or will correct for) so-called **systematic errors**, that is, effects that are present that may cause our results to be systematically (not randomly) high or low.

E.4 Assessing uncertainties and deviations from expected results

The primary reason for keeping track of measurement uncertainties is that the uncertainties tell us how much confidence we should have in the results of the measurements.

If the results of our measurements are compared to results expected on the basis of theoretical calculations or on the basis of previous experiments, we expect that, if no mistakes have been made, the results should agree with each other within the combined uncertainties. For the uncertainty we traditionally use the 95% Confidence Range (that is, two times the standard deviation).

(Note that even a theoretical calculation may have an uncertainty associated with it because there may be uncertainties in some of the numerical quantities used in the calculation or various mathematical approximations may have been used in reaching the result.) As a rule of thumb, if the measured results agree with the expected results within the combined uncertainties, we usually can view the agreement as satisfactory. If the results disagree by more than the combined uncertainties, something interesting is going on and further examination is necessary.

Example Suppose a theorist from MIT predicts that the value of X in some experiment to be 333 ± 1 Nm/s. Suppose that an initial experiment gives the result 339 ± 7 Nm/s, which result overlaps the theoretical prediction within the combined uncertainties. Hence, we conclude that there is satisfactory agreement between the measured value and the predicted value given the experimental and theoretical uncertainties. However, suppose that we refine our measurement technique and get a new result 340.1 ± 0.1 Nm/s. Now the measured result and the theoretical result do not agree. [Note that our new measured result is perfectly consistent with our previous result with its somewhat larger uncertainty.] We cannot tell which is right or which is wrong without further investigation and comparison.

E.5 Propagating uncertainties

(Sometimes mistakenly called "propagation of errors") In most measurements, some calculation is necessary to link the measured quantities to the desired result. The question then naturally arises: How do the uncertainties in the measured quantities affect (propagate to) the results? In other words, how do we estimate the uncertainty in the desired result from the uncertainties in the measured quantities?

E.5.1 "High-low Method"

One way to do this is to carry through the calculation using the extreme values of the measured quantities, for example 5.06 cm and 5.18 cm from the distance measurement example above, to find the range of result values. This method is straightforward but quickly becomes tedious if several variables are involved.

Example Suppose that you wish to determine a quantity, X, which is to be calculated indirectly using the measurements of a, b, and c, together with a theoretical expression: $X = \frac{ab}{c}$.

Suppose, further, that you have already determined that

$$\begin{aligned} a &= 23.5 \pm 0.2 \text{ m} \\ b &= 116.3 \pm 1.1 \text{ N} \\ c &= 8.05 \pm 0.03 \text{ s} \end{aligned}$$

The "best" value of X is computed from the best (mean) values of a, b and c:

$$X_{\text{best}} = \frac{23.5 \times 116.3}{8.05} = 339.509\,\text{Nm/s} \tag{34}$$

(We'll clean up the significant figures later.) But X could be about as large as what you get by using the maximum values of and b and the minimum (why?) value of c:

$$X_{\text{high}} = \frac{23.7 \times 117.4}{8.02} = 346.930\,\text{Nm/s} \tag{35}$$

And similarly, we find

$$X_{\text{low}} = \frac{23.3 \times 115.2}{8.08} = 332.198\,\text{Nm/s} \tag{36}$$

Notice that X_{high} and X_{low} differ from X_{best} by about the same amount, namely 7.3. Also note that it would by silly to give six significant figures for X. Common sense suggests reporting the value of X as, say, $X = 339.5 \pm 7.3$ Nm/s or $X = 339 \pm 7$ Nm/s.

E.5.2 General Method

The general treatment of the propagation of uncertainties is given in detail in texts on the statistical analysis of experimental data. Taylor's book [1] is a particularly good reference at this level. Here we will develop a very simple, but general method for finding the effects of uncertainties.

Suppose we want to calculate some result R, which depends on the values of several measured quantities x, y, z:

$$R = f(x, y, z) \tag{37}$$

Let us also suppose that we know the mean values and the uncertainties (standard deviations, for example) for each of these quantities. Then the uncertainty in R due to the uncertainty in x, for example, is calculated from

$$\Delta_x R = f(\overline{x} + \Delta\overline{x}, \overline{y}, \overline{z}) - f(\overline{x}, \overline{y}, \overline{z}) \tag{38}$$

where the subscript on Δ reminds us that we are calculating the effect due to x alone. We might call this the "partial uncertainty." Note that Eq. 38 is much like our "high-low" method except that we focus on the effect of just one of the variables. In a similar fashion, we may calculate the partial uncertainties in R due to Δy and to Δz.

By calculating each of these contributions to the uncertainty individually, we can find out which of the variables has the largest effect on the uncertainty of our final result. If we want to improve the experiment, we then know how to direct our efforts.

We now need to combine the individual contributions to get the overall uncertainty in the result. The usual argument is the following: If we *assume* that the measurements of the variables are independent so that variations in one do not affect the variations in the others, then we argue that the net uncertainty is calculated as the square root of the sum of the squares of the individual contributions:

$$\Delta R = \sqrt{(\Delta_x R)^2 + (\Delta_y R)^2 + (\Delta_z R)^2} \tag{39}$$

The formal justification of this statement comes from the theory of statistical distributions and assumes that the distribution of successive measurement values is described by the so-called **Gaussian** (or, equivalently, **normal**) distribution.

Note that our general method applies no matter what the functional relationship between R and the various measured quantities. It is not restricted to additive and multiplicative relationships as are the usual simple rules for handling uncertainties.

The method introduced here is actually just a simple approximation to a method that uses partial derivatives. Recall that in computing partial derivatives, we treat as constants all the variables except the one with respect to which we are taking the derivative. For example, to find the contribution of x to the uncertainty in R, we calculate

$$\Delta_x R = \frac{\partial f(x,y,z)}{\partial x}\Delta x \tag{40}$$

with analogous expressions for the effects of y and z. We then combine the individual contributions as above.

Example Suppose we have made some measurements of a mass m, a distance r, and a frequency f, with the following results for the means and standard deviations of the measured quantities:

$$\begin{aligned} m &= 150.2 \pm 0.1 \\ r &= 5.80 \pm 0.02 \\ f &= 52.3 \pm 0.4 \end{aligned}$$

(Note that we have omitted the units and hence lose 5 points on our lab report.)

From these measured values we want to determine the "best value" and uncertainty for the following computed quantity: $F = mrf^2$. The "best value" is computed by simply using the best values of m, r, and f: $F = 2382875.2$ (We'll tidy up the number of significant figures later on.)

Let's use our partial derivative method to find the uncertainty. (As an exercise, you should compute the contributions to the uncertainty in F using the finite difference method of Eq. 38.) First, let's determine the effect due to m:

$$\Delta_m F = \frac{\partial F}{\partial m}\Delta m = rf^2\Delta m = 1586. \tag{41}$$

Next, we look at the effect of r:

$$\Delta_r F = \frac{\partial F}{\partial r}\Delta r = mf^2\Delta r = 8217. \tag{42}$$

And finally, the effect of f is given by

$$\Delta_f F = \frac{\partial F}{\partial f}\Delta f = 2mrf\Delta f = 36449. \tag{43}$$

We see immediately that the measurement of f has the largest effect on the uncertainty of F. If we wanted to decrease the uncertainty of our results, we ought to work hardest at decreasing the uncertainty in f.

Finally, let's combine the uncertainties using the "square-root-of-the-sum-of-the-squares" method. From that computation we find that we ought to give F in the following form:

$$F = (2.383 \pm 0.037) \times 10^6 \tag{44}$$

or

$$F = (2.38 \pm 0.04) \times 10^6 \tag{45}$$

in the appropriate units. Note that we have adjusted the number of significant figures to conform to the stated uncertainty. As mentioned above, for most purposes, citing the uncertainty itself to one significant figure is adequate.

Exercise 2 Use the "high-low" method to find the uncertainty in F for the previous example. Compare that result to that obtained by the partial derivative method. What are the advantages and disadvantages of the two methods?

E.5.3 Connection to the traditional simple rules for uncertainties

To see where the usual rules for combining uncertainties come from, let's look at a simple functional form:

$$R = x + y \tag{46}$$

Using our procedure developed above, we find that

$$\Delta_x R = \Delta x$$
$$\Delta_y R = \Delta y$$

and combining uncertainties yields

$$\Delta R = \sqrt{(\Delta x)^2 + (\Delta y)^2}.$$

The traditional rule for handling an additive relationships says that we should add the two (absolute) uncertainty contributions. We see that the traditional method overestimates the uncertainty to some extent.

Exercise Work out the result for a multiplicative functional relationship $R = f(x, y) = xy$. Compare our method with the traditional method of "adding relative uncertainties" for multiplicative relationships. (See the following section if you get stuck.)

E.6 Simplified Uncertainty Rules

E.6.1 For a sum

Add the absolute uncertainties, i.e.

$$\text{If } A = B + C \text{ then } \Delta A = \Delta B + \Delta C \tag{47}$$

E.6.2 For a difference

Add the absolute uncertainties, i.e.

$$\text{If } A = B - C \text{ then } \Delta A = \Delta B + \Delta C \tag{48}$$

E.6.3 For a product

Add the relative uncertainties, i.e.

$$\text{If } A = B \times C \text{ then } \frac{\Delta A}{A} = \frac{\Delta B}{B} + \frac{\Delta C}{C} \tag{49}$$

E.6.4 For a ratio

Add the relative uncertainties, i.e.

$$\text{If } A = \frac{B}{C} \text{ then } \frac{\Delta A}{A} = \frac{\Delta B}{B} + \frac{\Delta C}{C} \tag{50}$$

E.6.5 For multiplication by a constant

Multiply uncertainty by constant, i.e.

$$\text{lf } A = kB \text{ then } \Delta A = k\Delta B \tag{51}$$

E.6.6 For a square root

Divide the relative uncertainty by 2.

$$\text{If } A = \sqrt{B} \text{ then } \frac{\Delta A}{A} = \frac{1}{2}\frac{\Delta B}{B} \tag{52}$$

E.6.7 For powers

Multiply relative uncertainty by power, i.e.

$$\text{lf } A = B^n \text{ then } \frac{\Delta A}{A} = |n|\frac{\Delta B}{B} \tag{53}$$

E.6.8 For functions

Differentiate the function, i.e.

$$\text{lf } A = A(x) \text{ then } \Delta A = \left|\frac{dA}{dx}\right|\Delta x \tag{54}$$

References

[1] J. R. Taylor. *An Introduction to Error Analysis*. University Science Books, Sausalito, California, second edition, 1997.

[2] P. R. Bevington and D. K. Robinson. *Data Reduction and Error Analysis for the Physical Sciences*. McGraw-Hill, New York, second edition, 1992.