Physics 30 – Midterm 2 – Spring 2010

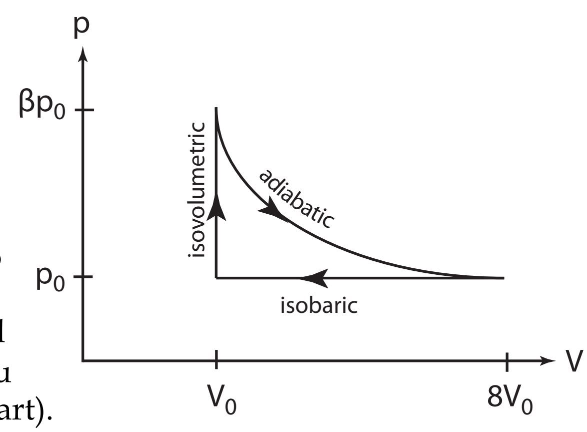
This is a take-home, self-scheduled **4 hour** exam.

You may use your textbook, your notes, past problem sets and problem set solution (posted on the course website). You may **not** use other textbooks or internet resources (though they're unlikely to help you anyway). You may use graphing calculators or software (such as Mathematica). You may use Mathematica or Mathematica-like software as an arithmetic aid, but do not print out or blindly copy machine-generated results, since they are usually practically unreadable to humans.

p-V loop (50 points)

A monatomic ideal gas undergoes a quasistatic three-stage process in p-V space.

1. Find β.
2. Calculate the heat absorbed by the gas (Q), the work done on the gas (W), the change in internal energy (ΔU) and the change in entropy (ΔS) for each leg of the process. Give your answer in terms of p_0V_0. Please label your results clearly on the exam so I don't have to guess which result attaches to which leg of the process.
3. For the entire loop, verify that $\Delta U_{total}=0$ and that $\Delta S_{total}=0$. (This is a good check that you haven't made a sign error in the previous part).
4. Calculate the efficiency of this engine.



Adsorption to a surface (30 points)

An ideal gas of Argon atoms, at room temperature, is adsorbing onto a surface. The binding energy between an Ar atom and the surface is ε = 10 kT. If the Argon gas has a concentration of 10^{-3} moles/liter, what fraction of the adsorption sites on the surface will be occupied by Ar atoms at equilibrium?

You may assume that the fraction is small and that the number of atoms adsorbed is negligible compared to the total number of atoms in the gas phase.[1]

[1] Neither of these assumptions is critical to the physics, but they make the algebra simpler.

1. Calculate the multiplicity for the adsorbed phase of n atoms adsorbed onto N sites.
2. Write down the free energy for the adsorbed phase.
3. Calculate the chemical potential of the adsorbed phase. N.B. $d(\ln n!)/dn \sim \ln n$.
4. Set the vapor / adsorption equilibrium condition using chemical potentials and solve for n/N.[2]

To reduce the amount of calculator work required, you may scale V_{Ar} from the quantum volume for *hydrogen* at room temperature, which is

$$V_H = \lambda_H^3 = \left(\frac{h^2}{2\pi m_H kT}\right)^{3/2} = \left(1.01\ \text{Å}\right)^3 .$$

Triple point (20 points)

A substance of molecular weight μ (g/mole) has its triple point at the absolute temperature T_0 and pressure p_0. At the triple point the densities of the solid and liquid are ρ_s and ρ_l, while the vapor is an ideal gas. At the triple point, the slope of the melting curve is $(dp/dT)_{ls}$ and the slope of the boiling curve is $(dp/dT)_{gl}$.

1. In terms of the quantities defined above, what is the slope of the sublimation curve $(dp/dT)_{gs}$?

Your answer should simplify considerably in terms of the non-dimensional quantities

$$\alpha \equiv \frac{\rho_l R T_0}{\mu p_0} \text{ and } \beta \equiv \rho_l/\rho_s$$

2. For water at the triple point, what are α and β? Given these α and β, is your expression in part 1 consistent with the phase diagram on p. 168 of the textbook? At the triple point, $\rho_l = 1.00\ \text{g/cm}^3$, $\rho_s = 0.92\ \text{g/cm}^3$, $p_0 = 611\ \text{Pa} = 611\ \text{N/m}^2$ and $T_0 = 273$ K.

[2] Congratulations! You've just constructed the Langmuir theory of adsorption.