

**Math 111, Introduction to the Calculus, Fall 2011**  
**Midterm III Practice Exam 1**

You will have 50 minutes for the exam and are not allowed to use books, notes or calculators. Each question is worth 10 points.

1. Find the critical points of the function

$$f(x) = x^2(x + 1)^2$$

For each critical point, decide if it is a local maximum or local minimum, or neither.

2. (a) Show that the function

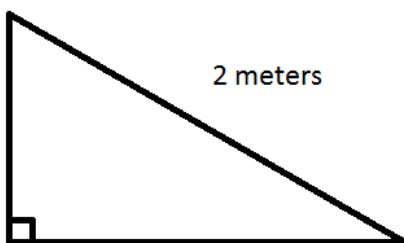
$$f(x) = x^3 + 6x$$

is increasing on all of  $\mathbb{R}$ .

- (b) Find the intervals on which  $f(x)$  is concave up or concave down, and find the inflection points.

- (c) Sketch a graph of  $f(x)$  based on the information from parts (a) and (b).

3. A right-angled triangle has hypotenuse 2 meters, as in the diagram below:



What is the largest possible area of the triangle?

4. Calculate each of the following integrals:

(a)  $\int_0^2 x^2 dx$ ;

(b)  $\int_1^2 x^{-2}(\sqrt{x} + 1) dx$ ;

(c)  $\int_0^{\frac{\pi}{2}} \sin(2t) dt$ .

5. A bathtub holds 54 gallons of water. If the tub is filled at a rate of  $3t$  gallons per minute at the time  $t$  minutes after starting, how long does it take to fill up the whole tub? Explain your answer fully.