Amherst College
Economics 66

The Basic Mathematics of Gary Becker's

Crime and Punishment

Basic Model

Becker is concerned with exploring the characteristics of optimal deterrence policy. He uses the following notation:

O = Number of offenses
p = Probability of apprehension
f = Monetary value of cost paid when apprehended.

The goal of deterrence policy is to choose p and f to minimize total costs of crime

Total cost has three components:

$D(O)$ = Damages of crime (as function of offenses)
$C(O,p)$ = Costs of apprehension (as function of offenses and probability of apprehension)
Expected penalty cost = $bpfO$ where b translates into social costs of penalty ($b \approx 0$ for fines).

Total social cost = $D(O) + C(O, p) + bfpO$.

Optimal *f*

In differentiating this must recognize that $O = O(p,f)$

$$\frac{\partial TC}{\partial f} = D'O_f + C'O_f + bpO + bpfO_f = 0$$

Dividing this by O_f and manipulating the equation gives:

$$D' + C' = -bpf(1 + \frac{1}{e_{O,f}})$$

Where $e_{O,f}$ is the elasticity of O with respect to f. In more familiar terms, this elasticity is the deterrent effect of monetary criminal sanctions.

Notice that the equation favors greater deterrence if $b = 0$ than if $b > 0$. With costly sanctions some degree of under-deterrence is optimal.

Since D' and C' are both positive, this optimality equation can only be satisfied if $-1 < e_{O,f} < 0$. In words, the supply of offenses must be inelastic with respect to *f*.

Generally this would require increasing *f* until all the "easy to deter crimes" are eliminated.

Optimal *p*

Optimal p is found by differentiation also. Can show that optimality requires $e_{O,p} < e_{O,f}$. That is, supply of offenses should be more elastic with respect to variations in *p* than with respect to variations in *f*. That is, penalty should be pushed further into the inelastic range than should apprehensions. This reflects the higher social costs of apprehension.

Behavioral problem with Becker conclusion

If criminals are risk averse, the elasticities have the opposite relationship to what is optimal.

Proof using logarithmic utility

$$E(U) = pU(W - f) + (1 - p)U(W) = p\ln(W - f) + (1 - p)\ln(W)$$
$$\frac{\partial U}{\partial p} = \ln(w - f) - \ln(W) \approx \frac{-f}{W}$$
$$\frac{\partial U}{\partial p} \cdot \frac{p}{U} = \frac{-pf}{UW}$$

and

$$\frac{\partial U}{\partial f} = \frac{-p}{(W - f)}$$
$$\frac{\partial U}{\partial f} \cdot \frac{f}{U} = \frac{-pf}{U(W - f)}$$

Hence, it is clear that the elasticity of utility with respect to *f* is more negative than the elasticity of *U* with respect to *p*. This is true for any risk adverse utility function (such as the logarithmic). Hence, although the data (see Cornwall and Trumball) tend to show that $e_{O,p} << e_{O,f}$ (as is socially optimal), there is uncertainty about the conclusions with regard to criminal behavior to be drawn from this finding. It may be that existing empirical studies do not capture all that is relevant about criminal behavior.