Amherst College
Economics 58
Fall, 2010 **Name______________**

Examination # 4

Solutions

There are three questions on this 60 minute examination. Each is of equal weight in grading.

1. Suppose that the demand for stuffed koalas is given by:

Q = 100 – P

Where Q is the quantity of koalas bought each year and P is the price of this fabulous item. Suppose also that stuffed koalas can be produced at a constant marginal and average cost of 40 per bear.

a. If there is a single monopoly supplier to the stuffed koala market, what quantity will this producer choose and what price will be charged?

$$\pi = PQ - 40Q = 100Q - Q^2 - 40Q$$
$$\frac{d\pi}{dQ} = 0 = 60 - 2Q \Rightarrow Q = 30, P = 70$$

b. Given the equilibrium described in part a, what will be the value of monopoly profits, consumer surplus, and the deadweight loss (relative to a situation where stuffed koalas are produced competitively at a price of 40).

$$\pi = 30(70 - 40) = 900$$
$$CS = 0.5(100 - 70) \bullet 30 = 450$$
$$CS_{competitive} = 0.5(100 - 40)60 = 1800$$
$$DWL = 1800 - 900 - 450 = 450 = 0.5(70 - 40)30$$

c. Suppose that a second stuffed koala producer (also with average and marginal costs of 40) enters the market. Calculate the Cournot/Nash equilibrium that will prevail in this market. (that is, the two producers compete in a Cournot quantity-setting game)

$\pi_1 = Pq_1 - 40q_1 = 100q_1 - q_1^2 - q_1q_2 - 40q_1$

$\frac{\partial \pi_1}{\partial q_1} = 60 - 2q_1 - q_2 \Rightarrow q_1 = \frac{60 - q_2}{2}$

Symmetry implies $q_2 = \frac{60 - q_1}{2}$

Solving simultaneously gives $q_1 = q_2 = 20, P = 60$

d. Calculate the firms' total profits at the Cournot equilibrium. Explain in general terms the conditions under which these firms might be able to maintain monopoly profits in this market if this game is played many times.

$\pi_1 = \pi_2 = (60 - 40)20 = 400$

Total Profits = 800

A trigger strategy will work here to maintain profits of 900 (rather than 800) providing the discount rate is low enough.

2. (A question in the spirit of the season). Kris owns a Christmas tree farm. Each of his trees grows according to the exponential equation $Height(t) = e^{\sqrt{t} - .05t}$. Consumers are always willing to pay exactly $1 per foot for a Christmas tree. The real interest rate is 5 percent. (Hint: Throughout this problem you should use continuous compounding – you may leave answers in terms of *e* or work out an exact number if you have a calculator)

a. If Kris wishes to maximize the height of his trees, how long should he let them grow? How tall will the trees be if Kris follows this plan?

$$\frac{dH(t)}{dt} = \frac{1}{2\sqrt{t}} - .05 = 0 \Rightarrow t = 100, H(100) = e^5.$$

b. Explain why the plan outlined in part a is not optimal for Kris. How long should he let his trees grow to maximize his returns? How tall will trees be if Kris adopts an optimal plan? What will each tree sell for?

Tree's growth slows down. Kris would not want the growth rate to fall below five percent. Can optimize by maximizing PDV of tree:

$$PDV = e^{-.05t}H(t) = e^{\sqrt{t} - 0.1t}$$

$$\frac{dPDV}{dt} = \frac{1}{2\sqrt{t}} - 0.1 = 0 \Rightarrow t = 25, H(25) = e^{3.75}$$

c. A 15 year old tree is 22.7 feet tall. Will this un-harvested tree sell for $22.70? If not, what price will the tree sell for?

A 15 year old tree is worth more if it is left growing because it is growing faster than 5 percent per year. Value at year 15 is equal to value at year 25 discounted by ten years' interest:

$$e^{3.75}e^{-0.05(10)} = e^{3.25} > 22.70$$

e. Without making any explicit calculations, explain how you would expect an increase in *r* to affect the length of time Kris allows a tree to grow.

An increase in *r* will cause the tree to be cut down sooner because its growth rate is always declining.

3. The following questions refer to Weitzman's review of the Stern report (note there are only four parts to this question).

a. On page 706 Weitzman refers to Equation 1 as the "Ramsey equation". Explain precisely how this equation relates to the "Euler Equation" presented in class. Specifically, how does the parameter η in the equation relate to the parameter ρ in the relative CRRA utility function $U(W) = W^{\rho}/\rho$. Why does he refer to this as "the elasticity of marginal utility"? Why does this measure the "aversion to <u>inter-temporal</u> inequality"? (note the typo in the text in the right hand column of page 706)

With this utility function $U'(W) = W^{\rho-1}$ so $\rho - 1$ is the elasticity of marginal utility. Weitzman used the absolute value of this, so $\eta = 1 - \rho$.

b. Weitzman shows explicitly the importance of an assumed rate of return on page 708 where he calculates that the benefit/cost ratio of spending 1% of GDP now to avoid a 5% reduction in 100 years is given by $\frac{B}{C} = 5e^{100(g-r)}$. Explain how Weitzman derives this equation. Explain also the equation's relevance to the global warming debate.

$$C = .01GDP_0 \quad B = .05GDP_{100} = .05GDP_0 e^{100g}$$
$$PDVB = .05GDP_{100}e^{-100r} = .05GDP_0 e^{100(g-r)}$$
$$\frac{PDVB}{C} = \frac{.05GDP_0 e^{100(g-r)}}{.01GDP_0} = 5e^{100(g-r)}$$

c. Weitzman writes, "The upshot of this uncertainty about uncertainty is that the reduced-form probability distribution of g … has a thick left tail. The exact thickness of this left tail of g depends not only upon how bad an environmental catastrophe global warning might induce and with what probabilities, but also upon how imprecise our probability estimates of the probabilities of those bad catastrophes." (middle of p.718)

Consider the following simple model: Suppose that society has a utility function for which the utility derived from a certain economic growth rate is given by the growth rate itself: $U(g) = g$. But suppose that economic growth is uncertain because there is some probability (*p*) the economy will suffer a catastrophe due to climate change. If the normal growth rate is μ, this rate falls to $\mu - \Delta$ with probability *p* and stays at μ with probability (1-*p*). In this model, "thickness of the left tail" refers to the perceived value of *p*. Assume also that society maximizes the following expected utility with respect to economic growth: E(U(g)) = E(g) – 40 Var(g). What is the expected value of utility given the risks being faced in this situation? [Hint: remember that the variance of a simple binomial variable (0,1) is $Var(x) = p(1-p)$ -- in this case the usual binomial is multiplied by Δ.]

$$E(g) = (1-p)\mu + p(\mu - \Delta) = \mu - p\Delta$$
$$Var(g) = \Delta^2 p(1-p)$$
$$U(g) = \mu - p\Delta - 40\Delta^2 p(1-p)$$

d. Suppose that society can achieve a certain growth rate of $\mu - p\Delta - \beta$ by undertaking environmental investments that reduce the expected growth rate by β. What is the highest value of β for which society is willing to exercise this option, in terms of $\{\mu, \Delta, p\}$? Does this maximum β increase or decrease with the "thickness of the left tail" (that is with *p*)?

$$\mu - p\Delta - \beta = \mu - p\Delta - 40\Delta^2 p(1-p) \Rightarrow$$
$$\beta = 40\Delta^2 p(1-p)$$
$$\frac{d\beta}{dp} = 40\Delta^2(1-2p) > 0 \text{ for small } p.$$

Example:

$$\Delta = 0.05, p = .01 \Rightarrow \beta = 40(.0025)(.0099) \approx 0.001$$
$$\Delta = 0.05, p = .05 \Rightarrow \beta = 40(.0025)(.0475) \approx 0.005$$