Amherst College
Economics 58
Fall 2010

Solutions to PS # 6

1. a. $q = 2\sqrt{kl}$, $k = 100, q = 2\sqrt{100\,l}$ $q = 20\sqrt{l}$

$$\sqrt{l} = \frac{q}{20} \quad l = \frac{q^2}{400}$$

$$SC = vK + wL = 1(100) + 4\left(\frac{q^2}{400}\right) = 100 + \frac{q^2}{100}$$

$$SAC = \frac{SC}{q} = \frac{100}{q} + \frac{q}{100}$$

b. $SMC = \frac{q}{50}$. If $q = 25, SC = 100 + \left(\frac{25^2}{100}\right) = 106.25$

$$SAC = \frac{100}{25} + \frac{25}{100} = 4.25 \quad SMC = \frac{25}{50} = .50$$

If $q = 50$, $SC = 100 + \left(\frac{50^2}{100}\right) = 125$

$$SAC = \frac{100}{50} + \frac{50}{100} = 2.50 \quad SMC = \frac{50}{50} = 1$$

If $q = 100$, $SC = 100 + \left(\frac{100^2}{100}\right) = 200$

$$SAC = \frac{100}{100} + \frac{100}{100} = 2 \quad SMC = \frac{100}{50} = 2.$$

If $q = 200$, $SC = 100 + \left(\frac{200^2}{100}\right) = 500$

$$SAC = \frac{100}{200} + \frac{200}{100} = 2.50 \quad SMC = \frac{200}{50} = 4.$$

c. [Figure 10.5]

d. As long as the marginal cost of producing one more unit is below the average-cost curve, average costs will be falling. Similarly, if the marginal cost of producing one more unit is higher than the average cost, then average costs will be rising. Therefore, the *SMC* curve must intersect the *SAC* curve at its lowest point.

e. $q = 2\sqrt{\bar{k}l}$ so $q^2 = 4\bar{k}l$ $\quad l = q^2 / 4\bar{k}$

$SC = v\bar{k} + wl = v\bar{k} + wq^2/4\bar{k}$

f. $\frac{\partial SC}{\partial k} = v - wq^2/4\bar{k}^{-2} = 0$ so $\bar{k} = 0.5qw^{0.5}v^{-0.5}$

g. $C = vk + wl = 0.5q\, w^{0.5}v^{0.5} + 0.5q\, w^{0.5}\, v^{0.5} = qw^{0.5}\, v^{0.5}$ (a special case of Example10.2)

h. If $w = 4$ $\quad v = 1$, $\quad C = 2q$

$SC = (\bar{k} = 100) = 100 + q^2/100$, $SC = 200 = C$ for $q = 100$

$SC = (\bar{k} = 200) = 200 + q^2/200$, $SC = 400 = C$ for $q = 200$

$SC = 800 = C$ for $q = 400$

2. As for many proofs involving duality, this one can be algebraically messy unless one sees the trick. Here the trick is to let $B = (v^{.5} + w^{.5})$. With this notation, $C = B^2q$.

a. Using Shephard's lemma,

$$k = \frac{\partial C}{\partial v} = Bv^{-0.5}q \quad l = \frac{\partial C}{\partial w} = Bw^{-0.5}q.$$

b. From part a,

$$\frac{q}{k} = \frac{v^{0.5}}{B}, \quad \frac{q}{l} = \frac{w^{0.5}}{B} \quad so \quad \frac{q}{k} + \frac{q}{l} = 1 \quad or \quad k^{-1} + l^{-1} = q^{-1}$$

The production function then is $q = (k^{-1} + l^{-1})^{-1}$.

b. This is a CES production function with ρ = -1. Hence, σ = 1/(1-ρ) = 0.5. Comparison to Example 10.2 shows the relationship between the parameters of the *CES* production function and its related cost function.

3. a. In each case, multiplying the inputs by t will multiply output by t^s.

b. **Fixed Proportions:** Cost minimization requires $k = l = q^{1/s}$. Total costs are $C(q,v,w) = vk + wl = l(v+w) = q^{1/s}(v+w)$.

Perfect Substitutes: Assume $w < v$. Cost minimization requires $l = q^{1/s}, \quad k = 0$. Total costs are $C(q,v,w) = v(0) + wl = q^{1/s}w$. Since a similar argument would hold when $v < w$, we conclude that in all cases, $C(q,v,w) = q^{1/s}[Min(v,w)]$.

c. Yes, both of these are separable.

d. For any separable cost function

$AC = \frac{f(q)}{q}C(1,v,w) \quad MC = f'(q)C(1,v,w)$. In these cases:

Fixed Proportions: $AC = q^{(1-s)/s}(v+w) \quad MC = s^{-1}q^{(1-s)/s}(v+w)$.

Perfect Substitutes:
$AC = q^{(1-s)/s}[Min(v,w)] \quad MC = s^{-1}q^{(1-s)/s}[Min(v,w)]$.