

### Elasticity Calculations

Making predictions about market responses is especially easy if you use constant elasticity demand and supply curves. This exercise asks you to make a few such calculations and compare them to intuitive/back-of-the-envelope calculations. Throughout assume that demand is given by  $Q_D = aP^b$  (where  $b$  is the price elasticity of demand) and supply by  $Q_S = cP^d$  (where  $d$  is the elasticity of supply and could be either a short-run or long-run figure).

a. Calculate the equilibrium price ( $P^*$ ) in this market.

$$P^* = \left(\frac{a}{c}\right)^{\frac{1}{d-b}}$$

b. Suppose that  $d = 0$ , calculate the effect of a one percent reduction in supply  $c' = 0.99c$  on equilibrium price in general (your answer should be in terms of  $P^*$ ).

$$\text{Price will change by } P = P^* \left(\frac{1}{0.99}\right)^{\frac{1}{-b}} = P^* (1.01)^{1/-b}$$

c. Use your results from b. to calculate the precise proportional increase in price for each of the following values of  $b$ :

$$b = -1 \quad P = P^* (1.01)$$

$$b = -0.3 \quad P = P^* (1.01)^{3.33} = P^* (1.0337)$$

$$b = -1.7 \quad P = P^* (1.01)^{0.588} = P^* (1.0059)$$

d. How do your results compare to an intuitive calculation for each of these cases?

Case 1 would predict a 1% rise in price

Case 2 would predict a 3.3% rise in price

Case 3 would predict a 0.59% rise in price

So the intuitive calculations are quite close.

e. Now calculate the effect on price of an arbitrary proportional reduction in supply of  $k$ . That is, now  $c' = (1-k)c$  (continue to assume  $d = 0$ ).

$$P = P^* \left( \frac{1}{1-k} \right)^{\frac{1}{-b}}$$

f. Use your results from e to fill in the following table which shows the percent increase in price for various values of  $k$  and  $b$ .

| Value of $k$ /Value of $b$ | $b = -0.3$ | $b = -1$ | $b = -1.7$ |
|----------------------------|------------|----------|------------|
| $k = 0.02$                 | 1.0696     | 1.0204   | 1.0119     |
| $k = 0.1$                  | 1.4202     | 1.1111   | 1.0639     |
| $k = 0.25$                 | 2.6062     | 1.3333   | 1.1844     |

g. What do your results in f. show?

Intuitive calculations are still pretty good for a 2% reduction in supply. Once the reduction gets bigger, the approximations get further from the mark.

h. Now calculate the effect of a proportional supply reduction of  $k$  on equilibrium price for the general case in which  $d \neq 0$ .

$$P = P^* \left( \frac{1}{1-k} \right)^{\frac{1}{d-b}}$$

i. Use your results from h to fill out the following two tables: One for  $d = 0.2$  and one for  $d = 1.0$ :

$d = 0.2$

| Value of $k$ /Value of $b$ | $b = -0.3$ | $b = -1$ | $b = -1.7$ |
|----------------------------|------------|----------|------------|
| $k = 0.02$                 | 1.0412     | 1.0170   | 1.0107     |
| $k = 0.1$                  | 1.2352     | 1.0917   | 1.0570     |
| $k = 0.25$                 | 1.7777     | 1.2708   | 1.1634     |

$$d = 1.0$$

| <i>Value of k/Value of b</i> | $b = -0.3$ | $b = -1$ | $b = -1.7$ |
|------------------------------|------------|----------|------------|
| $k = 0.02$                   | 1.0157     | 1.0101   | 1.0075     |
| $k = 0.1$                    | 1.0844     | 1.0541   | 1.0397     |
| $k = 0.25$                   | 1.2476     | 1.1547   | 1.1123     |

j. How do these results compare to intuitive calculations you might make?

Intuitive calculations are easiest for  $d = 1, b = -1$  in which case  $\frac{1}{d - b} = .5$ . Hence a percent shortfall in supply will raise price by one-half of one percent. This approximation works well for  $k = .02$  and not badly for  $k = 0.1$ . It misses a lot for  $k = 0.25$ , however.