Problem Session 1 for Math 29: Cruise Activities, Some Theory, and The Simpsons

1 Cruise Activities

You are trying to arrange a cruise for yourself and 24 of your closest friends. There are two cruise ships available. The first has room for 20 guests and the second has room for 30.

For the smaller cruise ship, how many different passenger manifests (list of passengers in no particular order, except usually just listed alphabetically) are there? (Note, you are not guaranteed a spot on the smaller cruise!)

Assuming that you guarantee yourself a spot and that 2 of the other passengers are people that you insist come with you, how many passenger manifests are there for the smaller ship?

You decide to pick the larger cruise ship and get ready to set off for 7 days/nights and are now dealing with food selections for dinner. The chef has a selection of 10 main entrees, 8 sides (you get two distinct sides per meal), and six desserts. How many different dinners are there? Do you think that's enough variety for everyone over 7 days?

After dinner one night, there is a movie double feature. 12 films are available to show (assume you had some input as to what those might be) and you won't play a movie more than once. How many different double features can be shown (A then B is different than B then A as you might experience movie A differently after seeing movie B) for that evening?

If there are 6 refreshments available and you can have 3 different ones out during the showing, how many feature/refreshment arrangements are there?

For the final night of the cruise, there is a series of contests ranging from Karaoke to limbo, with six contests total.

Assuming you can win repeated events, how many final winners lists (list of the 6 winners) are possible?

Assuming the events are done from highest to lowest difficulty and if you win you stop competing, how many winners lists are possible?

Assuming if you win you stop competing, but the winners lists no longer show events and just list winners alphabetically, how many winners lists are possible?

2 Dreamboat Cars

Three different factories (A,B,C) produce dreamboat cars. A makes 20 percent of the total output, B makes 50 percent, and C makes the rest. However, it is known that 5 percent of the cars produced at A are lemons, while 2 percent from B are lemons, and a whopping 10 percent of those from C are lemons.
a. Suppose you buy a Dreamboat car and it is a lemon. What is the probability it came from A?
b. Suppose you buy a Dreamboat car and it is not a lemon. What is the probability it came from B?

3 Liver Scan Pathology

Tests done to see if a normal liver scan can detect abnormal liver pathology have revealed the following: (Row = True Status, Column = Test Status)

True/Test	Normal	Abnormal	Total
Normal	54	32	86
Abnormal	27	231	258

Treat abnormal liver status as the "disease" or condition of interest.
a. What is the sensitivity of the normal liver scan for detecting abnormal liver pathology?
b. What is the specificity of the normal liver scan for detecting abnormal liver pathology?
c. If someone takes the test and has an abnormal liver scan as a result, what is the probability they actually have abnormal liver pathology?

4 Independence Day

Three events, A,B, and C, are said to be mutually independent if P(AB)=P(A)P(B), P(BC)=P(B)P(C), P(AC)=P(A)P(C), and P(ABC)=P(A)P(B)P(C). A and B are independent if P(AB)=P(A)P(B), and A and B are conditionally independent given C if P(AB given C)=P(A given C) P(B given C).

Assume you are playing with a single, fair, six-sided die, and the following events have been defined:
A: the first roll of a six-sided die is even
B: the second roll of a six-sided die is even
C: the sum of the two rolls is 7
Find $P(A), P(B), P(C)$, and any other relevant probabilities to classify the events as: mutually independent, pairwise independent, or conditionally independent (if conditionally independent, conditioned on which event).

Now assume you are playing with two different dice. One is a regular six-sided die and the other has had the $4, 5, 6$ replaced with another set of $1, 2, 3$. The first will be called the fair die and the second the unfair die. One die will be picked at random (equal probability of picking either die) and two rolls performed. The following events are defined:
A: the first roll is even
B: the second roll is even
C: the unfair die is rolled
Find $P(A), P(B), P(C)$, and any other relevant probabilities to classify the events as: mutually independent, pairwise independent, or conditionally independent (if conditionally independent, conditioned on which event). Note you may need to use the Law of Total Probability to compute some of the relevant probabilities.

5 The Simpsons - Not the TV show

For the year of 1973, the University of California at Berkeley was involved in legal action relating to the claim that there was gender discrimination in graduate school admissions (paper by Bickel, et.al. 1975 for details). The table below contains school-wide admission/denied numbers for 1973 for the pool of 12763 applicants.

	Admitted	Denied	Total
Male	3738	4704	8442
Female	1494	2827	4321

What is the probability a randomly selected applicant was female?
What is the probability a randomly selected applicant was admitted?
What is the probability a randomly selected applicant was admitted if you knew the applicant was female?
Do you think this is strong evidence of gender discrimination in the graduate school admission process?

Here is the breakdown by some of the graduate school departments (not all departments listed) (largest 6 departments out of 101 total departments). Fill in the remaining conditional probabilities of the form P(admit male given dept. A)= % M. Admit and P(admit female given dept. A)= % F. Admit for the few blanks.

Dept.	M. Admit.	M. Den.	M. Tot.	% M. Admit	F. Admit.	F. Den.	F. Tot.	% F. Admit
A	512	313	825		89	19	108	
B	313	207	560	.56	17	8	25	.68
C	120	205	325	.37	202	391	593	.34
D	138	279	417		131	244	375	
E	53	138	191	.28	94	299	393	.24
F	22	351	373	.06	24	317	341	.07

What do you think now about the claims of gender discrimination? What do you think was going on here?

Situations where probabilities seemingly "reverse" when a third variable (department) are taken into account are examples of a phenomenon known as Simpson's Paradox. In this case, it was determined that the "reversal" when department was taken into account was due to the fact that women had applied in larger numbers to departments that were harder to get into (fewer slots in proportion to number of applicants makes it harder even among qualified applicants) while men were applying to departments that were relatively easier to get into, resulting in a lower overall admission rate for women even if the department level bias appears to be in favor of women!