GENERAL EQUILIBRIUM MODELS:

IMPROVING THE MICROECONOMICS CLASSROOM

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The general equilibrium simulation program described in this paper is available at the following URL:

www.amherst.edu/~fwesthoff/compequ/FixedPointsCompEquApplet.html.

Abstract: General equilibrium modeling has come to play an important role in such fields as international trade, tax policy, environmental regulation, and economic development. Teaching about these models in intermediate microeconomics courses has not kept pace with these trends. The typical microeconomics course devotes only about a week to general equilibrium issues and microeconomics texts primarily focus on the insights that can be drawn from the Edgeworth Box diagram for exchange. We believe such treatment leaves students unprepared for understanding much of the policy-related literature they will encounter and, more generally, shortchanges their education in economics. In this paper, we illustrate how computer-based general equilibrium simulations might improve teaching about these topics in intermediate microeconomics courses. We provide several illustrations describing how simulations could be used to make important points about economic theory, public economics, and environmental regulation.

The use of general equilibrium modeling has expanded dramatically in recent years. Such models are now routinely employed to study tax incidence, environmental regulation, international trade impacts, and natural disasters. Insights provided by these modeling efforts could not have been obtained using standard partial equilibrium tools. Teaching about general equilibrium modeling has not generally kept pace with these developments, however. The standard intermediate microeconomics course coverage of general equilibrium concepts (if they are covered at all) spends about a week on the topic, mainly by introducing the Edgeworth Box diagram for exchange. Table 1 illustrates the type of coverage given to the central topics in general equilibrium theory in some of the leading microeconomics textbooks. Virtually all of the books discuss Pareto optimality, efficiency in production and exchange, and the "first fundamental theorem" of welfare economics. Few, if any, books cover general equilibrium modeling as it is practiced today.

We believe that this short-changing of general equilibrium concepts makes students ill prepared for understanding much current research. More generally, we believe that this lack of attention obscures some major economic principles that all students of economics should know. We begin by laying out some of the basic principles that we believe would be clarified by greater attention to general equilibrium models. We then describe a computer-based general equilibrium simulation program that may prove useful to instructors in making these points. The subsequent sections of the paper present several illustrations of how the simulation program works in practice and how it can provide insights beyond those obtainable from partial equilibrium analysis.

We use a "standard" Walrasian general equilibrium model composed of utility maximizing households and profit maximizing firms. The simulation uses a variant of the Scarf algorithm to find prices that clear each market. The underlying analytics of this general equilibrium model can be complex. The supply and demand equations for the goods and inputs tend to be complicated making it difficult, if not impossible, for even very good undergraduates to "sort things out." We believe that our simulation approach avoids this pitfall by utilizing the numerical results. The student can apply the basic qualitative microeconomic analysis to appreciate why the equilibrium prices changed as they did without delving into the complex general equilibrium analytics. For example, students can "see" how a tax on one consumption good affects not only the price of that consumption good, but also the price of other consumptions goods and inputs. While understanding the workings of the algorithm itself is no doubt far beyond reach of the undergraduate, the software allows him/her to use the algorithm to illustrate the ramifications of general equilibrium analysis.

INSIGHTS FROM GENERAL EQUILIBRIUM MODELS

The strongest reason for more extensive coverage of general equilibrium models in intermediate microeconomics courses is that such inclusion would offer many new insights to students about economics. Among those insights are:

- Prices (including prices for factors of production) are endogenous in market economies. The exogenous elements are household preferences, household endowments, and the productive technologies.
- Firms and factors of production are owned by individual households, either directly or indirectly. All firm revenue is ultimately claimed by some household.
- Governments are bound by budget constraints. Any model is incomplete if it does not specify how government receipts are used.
- The "bottom line" in all evaluations of policy options in economics is the utility of the individuals in society. Firms and governments are only intermediaries in getting to this final accounting.
- Lump-sum taxes have no incentive effects and provide "Pareto efficient" transfers. On the other hand, all "realistic" taxes produce incentive effects and are distorting, thereby raising important equity-efficiency distributional issues.

- There is a close tie between general equilibrium modeling and cost-benefit accounting. The general equilibrium approach helps to understand the distinction between productive activities and transfers and is necessary to get taxation, public good and externality accounting correct.

Unfortunately, many of these insights are either not mentioned or made more obscure by the way that general equilibrium is currently taught. For example, we doubt that focusing on how the Edgeworth exchange box is constructed helps students grasp how preferences actually affect relative prices. Similarly, showing how the production possibility frontier is developed from underlying production functions may obscure issues of input ownership and the overall budget constraints that characterize any economy.

A better approach, we believe, would be to introduce students directly to computer-based general equilibrium simulations. By showing how general equilibrium models are structured and by walking students through some sample computer simulations, all of the insights listed above should become more apparent. Doing this with existing software for general equilibrium modeling, however, may involve far more in set-up costs than the typical instructor wishes to incur. The computer-based general equilibrium simulation program described below is complex enough to give students a feel for how general equilibrium models work while at the same time being simple enough for students to understand its key elements.

COMPUTER-BASED GENERAL EQUILIBRIUM SIMULATION PROGRAM

Our general equilibrium simulation program follows the traditional Arrow-Debreu approach modified to include the possibility of taxes, a public good, and an externality. All households and firms act as price-takers. The simulation program has been coded in the Java language to provide a user-friendly interface. The program itself is very flexible and can accommodate an arbitrary number of goods, households, and firms. While the user can enter all the parameters of the economy (household endowments and utility functions; firm production functions; etc.), an alternative exists that most may find preferable. The user can open one of several existing files which specify the parameters for the illustrations that appear below. To reproduce our results, the user need only modify a few of the parameters such as the tax rates, the presence of a public good or externality, etc. In this way, the time and effort required to specify the parameters of the specific model can be minimized.

Households

The endowment of each good is specified for each household. Typically, each household is endowed with some non-produced goods (potential labor time and perhaps some capital) although any endowment scheme is permitted. A constant elasticity of substitution utility function is used to specify each household's preferences:

$$u(x_1, x_2, \ldots, x_G, P, R-E) = \left[\sum_{g=1}^{G} \alpha_g x_g^{\rho_C} + \alpha_P P^{\rho_C} + \alpha_P (R-E)^{\rho_C}\right]^{1/\rho_C}$$

where

G	=	Number of Private Goods
x_g	=	Quantity of Private Good g Consumed by the Household
P	=	Quantity of Public Good
R	=	Initial Quantity of "Resource"
E	=	Externality
α's	=	CES utility "coefficients"
ρ_C	=	$\frac{\sigma_C - 1}{\sigma_C}$ σ_C = Elasticity of Substitution Consumption

If a good provides no (direct) utility, as may be the case of capital, the value of α is set to zero. Furthermore, in the case of labor, the household may have a positive α indicating a household's preference to consume its endowed labor as leisure. The "externality term" requires explanation. We begin with a specified quantity of a resource and then the externality depletes the resource. For example, suppose there are 10 units of clean air available for the households to enjoy, but the external effect pollutes the air reducing the quantity of clean air available to the household. In this case, R would equal 10. For $\sigma_C = 1$, the utility functions take the simple Cobb-Douglas form. Each household is assumed to maximize its utility subject to a budget constraint that includes both consumer goods purchased and endowed resources sold.

Firms

A constant elasticity of substitution production function specifies the productive technology for each firm; the production function for a firm that produces good i is:

$$y_i = \beta_{ii}\left[\Sigma_{g \neq i}\, \beta_{ig}\, y_g^{\rho_P}\right]^{1/\rho_P}$$

where

y_i = Quantity of Good i Produced

y_g = Quantity of Input g Used ($g \neq i$)

β's = CES production "coefficients"

$\rho_P = \dfrac{\sigma_P - 1}{\sigma_P}$ σ_P = Elasticity of Substitution Production

For $\sigma_P = 1$, the production functions exhibit the simple Cobb-Douglas form. Firms are assumed to maximize profits. Because of the constant returns nature of the production technology, in equilibrium all firms earn zero long-run profits. Hence, it is the ownership of productive input endowments that provides incomes to consumers – there is no distinct "income" of firms. Consequently specification of firm ownership is unnecessary in the simulation program.

Price and Tax Conventions

All reported prices are the prices as seen by the households. For simplicity, the prices are normalized so as to sum to 1. *Ad valorem* taxes can be placed on any of the goods.[1] Because the reported prices are those seen by households, it is perhaps easiest to think of taxes as being legally incident on the firms even though the legal incidence of the tax is irrelevant. To clarify how taxes are modeled, consider two examples.

***Ad valorem* tax on an input**: Suppose that the price of labor were .40, then $P_L = .40$. Consider imposition of an *ad valorem* tax of .25 on labor input ($t_L = .25$). In this case, each household would receive .40 of income for each unit of labor supplied. The firm would be spending .50 for each unit of labor hired. The difference would go to the government as tax revenue. More generally, for each unit of labor "traded" the:

- household receives P_L of income;
- firm incurs $P_L(1 + t_L)$ of costs;
- government receives $P_L t_L$ of tax revenue.

***Ad valorem* tax on a consumption good**: Suppose that the price of consumption good X is .50, ($P_X = .50$) and the *ad valorem* tax on consumption good X is .10 ($t_X = .10$). Each household would spend .50 for each unit of consumption good X purchased. The firm would receive .45 for each unit of

[1] To allow for the possibility of a tax on the external effect, unit taxes can also be specified in the model. An *ad valorem* tax on the external effect would have no effect since, in the absence of government intervention, the "price" of the external effect is 0.

consumption good X sold. The difference would go the government as tax revenue. More generally, for each unit of consumption good X traded the:

- household pays P_X;
- firm receives $P_X(1 - t_X)$ of revenues;
- government receives $P_X t_X$ of tax revenue.

Government

A general equilibrium model allows us to explicitly account for the government's budget constraint. When the government collects tax revenue, something must be done with it. Broadly speaking, there are two choices:

- Redistribute the revenue as transfer payments to households
- Use the revenue to finance the production of public goods.

The simulation program allows us to specify "redistribution factors" that determines the portion of the government's tax revenue that is redistributed to each households. To satisfy the budget constraint the sum of the redistribution factors across households cannot total more than 1. If the sum totals less than 1, the portion of the tax revenue not redistributed will be used to finance the production of a public good.

Computational Procedure

Most consider Walras to be the founding father of general equilibrium analysis. It was Walras who proposed a tatonnement process that would lead each market in an economy to move toward equilibrium based on excess demand at any initial price configuration. Unfortunately, a procedure based on such a process does did not guarantee convergence; that is, algorithms based on tatonnement do not always succeed in finding an equilibrium. The pioneering work of Herbert Scarf in the late 1960's provided the alternative approach, however, which allowed the field of applied general equilibrium to develop (Scarf, 1967). Scarf's algorithm finds a vector of prices that are "approximate" equilibrium prices – approximate in the sense that at these prices, the market for each good is "nearly" in equilibrium: the quantity demanded differs from the quantity supplied by a small amount at most (Scarf, 1973). Our simulation program computes such equilibrium prices using Merrill's refinement of Scarf's algorithm (Merrill, 1971).

PREVIEW OF ILLUSTRATIONS

General equilibrium models allow the illustration of many important economic issues. We choose to present five here:

- Interconnectedness of Markets
- Equivalence of a General Consumption Tax and General Income Tax
- Lump Sum versus Distorting Taxes
- Financing Public Good Production and Efficiency
- Externalities and Efficiency

Our first two illustrations are designed to emphasize the fundamental general equilibrium principle of market interconnectedness. The first illustrates that the impact of a change in one market is not isolated to that particular market, but rather it affects other markets also. Second, we present a tax equivalence example to emphasize another illustration of market interconnectedness, the concept of circular flow. While circular flow is an integral part of macroeconomic courses, it is rarely mentioned in microeconomics. The third illustration provides a better appreciation of the natures of lump sum and distorting taxes by explicitly accounting for what the government does with the tax revenue it collects. In doing so, we clearly connect the concepts of tax distortions and Pareto optimality. The last two illustrations tackle more complicated issues, financing public goods and externalities, to show

how general equilibrium models can provide valuable insights that partial equilibrium analysis fails to capture fully.

In order to simplify our discussion, we include only a small number of goods, households, and firms in the specific general equilibrium model that we present below. Note that the simulation program itself is not limited in this regard. Also, for the sake of simplicity we specify Cobb-Douglas utility functions and Cobb-Douglas production functions. Specifically, we make the following designations:

Goods

Our model includes four goods: two consumption goods (X and Y) and two inputs (L and K). Throughout, L and K can be thought of as representing labor and capital respectively.

Households

To capture a diversity of consumers, two households are included possessing different utility functions and endowments:

		Household 1	Household 2
Utility Functions:		$U = X^{.5} Y^{.3} L^{.2}$	$U = X^{.4} Y^{.4} L^{.2}$
Endowments:	L	24	24
	K	40	10

The L appearing in the utility function represents leisure – it is the amount of the labor endowment that is not sold in the marketplace. In total, the two households are (arbitrarily) endowed with a total of 48 units of L, labor, and 50 units of K, capital. Since L appears in each household's utility functions, each household will "demand" some of its endowed labor to enjoy as leisure. Accordingly, there are two sources of demand for a household's labor, the household itself and firms. Note that while the households are endowed with identical amounts of labor, household 2 is endowed with more capital.

Firms

Firm 1 produces consumption good X and firm 2 consumption good Y. Each firm can be thought of as describing a competitive industry's production technology:

	Firm 1	Firm 2
Production Functions:	$X = L^{.8} K^{.2}$	$Y = L^{.2} K^{.8}$

The production of consumption good X is labor intensive and the production of consumption good Y is capital intensive.

With this background we now turn to our five illustrative examples.

ILLUSTRATON 1: INTERCONNECTEDNESS OF MARKETS

General equilibrium models allow us to account for the interconnectedness of markets; that is, the models recognize that changes in one market affect other markets also. To illustrate this we begin with a no-tax base case in Simulation 1 (see Table 2). Then, in Simulation 2, we impose an *ad valorem* tax of .40 on consumption good X. In this simulation, all tax revenue is redistributed to households; we have arbitrarily specified that half the tax revenue is redistributed to household 1 and half to household 2.[2] First, consider each simulation separately. In each case, the model has been solved for the competitive equilibrium; the quantity demanded equals the quantity supplied for all four goods. For example, in Simulation 1, the quantity of consumption good X demanded equals 15.70 plus 8.06 or 23.76, which just equals the quantity of consumption good X firm 1 produces. Similarly, the quantity of good Y demanded equals 13.51 plus 11.57, which just equals the quantity of consumption good Y firm 2 produces.

Now compare the two simulations. Not surprisingly, the X-Y price ratio (as viewed by households) increases from 1.4344 to 2.1725 when consumption good X is taxed. Similarly, the equilibrium quantity of consumption good X falls from 23.76 to 17.94. The connections between markets is illustrated first by the increase in Y consumption from 25.08 to 28.84. Notice also how the markets for the inputs are affected. The wage-rental rate decreases from 1.8243 to 1.5554. This results from the fact that the production of consumption good X is labor intensive; the tax on consumption good X depresses the wage rate relative to the capital rental rate.

The two simulations in Table 2 also illustrate the importance of accounting for the government's budget constraint. While the tax decreases the utility of household 1, it increases the utility of household 2. This occurs because the 3.40 of tax revenue is redistributed to the households on a 50-50 basis. With the transfer payment of 1.70, household 2 (who has a somewhat smaller relative preference for good X) finds itself better off even though consumption good X is now being taxed. Of course, a different set of redistribution factors or of preferences would result in different utility consequences for the households.

The standard partial equilibrium approach to analyzing a tax on a consumption good tax relies either on indifference curves and budget lines or on demand and supply curves. Neither of these standard approaches captures fully the impact of the tax, however. The indifference curve/budget line approach implicitly assumes that the prices of the non-taxed consumption goods and inputs (and therefore also income) remain constant; that is, with the exception of

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[2] We follow the standard general equilibrium practice of denoting production outputs with

the market for the taxed good, all other markets are assumed to be unaffected. The simulation shows how the indifference curve/budget line approach fails to capture the important consequences of market interconnectedness. Similarly, the standard demand/supply approach focuses on the "tax wedge" between the price paid by households and the price received by firms; households are hurt because they pay more and firms are hurt because they receive less. Subsequently, changes in consumer surplus and producer surplus are often calculated along with the excess burden. This analysis typically stops there. Little, if anything, is said about how changes in one market affect other output and input markets nor about how the tax revenues are used.

General equilibrium models address the deficiencies of the partial equilibrium approaches by illustrating how a tax on a single consumption good impacts the markets for other consumption goods and the markets for inputs. Just as households are affected by what happens in the market for the taxed good itself, they are also affected by what happens in these other markets. Also, the simulation explicitly shows that the government must do something with the tax revenue it collects. And what it ultimately does also affects household welfare. In this simulation, the government redistributes the tax revenue to households as transfer payments (in later simulations, we consider the production of public

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positive signs and inputs with negative signs.

goods). Firms and the government are intermediaries; ultimately, households bear all the ramifications of the tax because households own the firms and the factors of production. General equilibrium models allow these important principles to be clearly illustrated.

ILLUSTRATON 2: TAX EQUIVALENCE

Our model can be used to illustrate the well-know result of the equivalence between a general consumption tax and a general income tax (see, for example, Stiglitz, 2000, pp. 502-505). In Simulation 3 (see Table 3), both consumption goods, X and Y, are taxed at a rate of 20 percent. Both of the inputs are untaxed. In Simulation 4, the situation is reversed. Both sources of factor income are taxed at a rate of 25 percent, while the consumptions goods are untaxed. As shown in the table, the general tax on consumption goods is equivalent to the general tax on the sources of income, the inputs. The outcomes are identical in all respects. Notice in addition that labor supplied under both tax structures (L = 17.51) falls short of labor supply in the untaxed Simulation 1 (L = 15.09) – even commodity taxes have labor supply consequences. These observations reinforce the notion of the circular flow in the economy between products' and goods' markets. The tax equivalence example illustrates that, as a consequence of market interconnectedness, placing taxes at different points in the circle have identical microeconomic effects.

ILLUSTRATON 3: LUMP SUM VERSUS DISTORTING TAXES

With the exception of a head tax, lump sum taxes are not present in the real world; nevertheless, they can provide an instructive base case. However, because our simple specification of the economy lacks an inter-temporal aspect and capital does not enter the utility function of the households, the supply of capital is completely inelastic. While this is obviously unrealistic, it is useful because a tax on capital now provides us with a "lump sum" base case. On the other hand, a tax on labor is a "distorting" tax because labor endowments not provided to the market (leisure) enter the utility function of the households; consequently, the supply curve for labor is not completely inelastic.

A lump sum tax produces no substitution effects, only income effects. Therefore, when a lump-sum tax imposed, it is possible to redistribute the tax revenue back to the households as transfer payments in a way that keeps each household equally well-off. With a distorting tax, it is impossible to do this; even when all tax revenue is redistributed back to the households at least one household must find itself worse off. Typically, these principles are illustrated for the case of a single household by appealing to the standard utility maximizing diagram; the budget line is shifted in a parallel fashion for a lump sum tax and a non-parallel fashion for a distorting tax. General equilibrium models allow us to illustrate the principles in an alternative way with more than a single household. Focus attention on Simulations 1 and 5 appearing in Table 4. As before, Simulation 1 includes no taxes; on the other hand, Simulation 5 imposes a 50 percent tax on capital. These simulations confirm the assertion that in the context of our specific model, a tax on capital is a lump sum tax; that is, the tax on capital leads to no distortions. Such a tax does not affect the allocation of resources; each firm's production decisions are unaffected. The tax does not alter the X-Y price ratio either; hence, the slope of each household's budget constraint is unaffected. The tax on capital raises tax revenues of 2.37. When 80 percent of the revenues are redistributed to household 1 and the remaining 20 percent to household 2, both households have their endowments "restored" and are just as well off as they were in the no tax situation. This 50 percent general tax on capital is therefore a non-distorting, lump sum tax.

Comparison of Simulations 1 and 6 illustrate the impact of a distorting tax. The 50 percent *ad valore*m tax on labor in Simulation 6 affects the allocation of resources. When the 2.73 of tax revenues is redistributed by giving 56 percent to household 1 and 44 percent to household 2, household 1 is (almost) just as well off as in the no tax situation, but household 2 is worse off. It is impossible to redistribute the tax revenue so as to keep both households equally well-off. Consequently, the tax on labor is a distorting tax that results in a deadweight loss. The distribution of this deadweight loss will, however, depend of how the tax revenues are redistributed to the households.

We believe that a simulation including two (or more) households allow students to appreciate better the notion of tax distortions and their intimate relationship to the Pareto criterion. In the classroom, we typically illustrate a lump sum tax as an inward parallel shift in the budget line and then observe that that no substitution effect results. Subsequently, by showing that a distorting tax leads to a non-parallel shift, we conclude that it is the substitution effect which leads to tax distortions. This analysis is incomplete, however, because it ignores the fact that the government must do something with the tax revenue it collects; that is, tax revenue does not disappear into a "black hole" as the standard diagram suggests. In this illustration, all the tax revenue is redistributed to the households for the purpose of illustrating the distinction between lump sum and distorting taxes. By doing so, we connect the notion of tax distortions with the basic Pareto criterion. In the case of a lump sum tax, each household can be made just as well off by redistributing the tax revenue in just the right way. In the case of a distorting tax, at least one household is hurt regardless of how the tax revenue is redistributed.

ILLUSTRATION 4: FINANCING AND THE OPTIMAL LEVEL OF PUBLIC GOOD PRODUCTION

To simplify matters, we have modified our model to study the financing of public goods. First, only a single household is included. In this way, it is more straightforward to focus on efficiency, as there are no distributional effects. Second, we add the public good to the representative household's utility function:

$U = X^{.5}\, Y^{.3}\, L^{.2}\, G^{.1}$ where G = Public Good

The household's endowments are unchanged, but the tax revenue collected is not redistributed; instead, it is used to finance the public good. The production functions for the consumption goods, X and Y, are unchanged. The production function for the public good takes the simple Cobb-Douglas form:

$G = L^{.5}\, K^{.5}$

Table 5 reports on two schemes to finance the production of the public good. The first uses a tax on capital, a lump sum tax in our model, to finance production and the second a tax on labor, a distorting tax. In each case, the quantity of the public good financed and the resulting level of household utility are reported for selected capital and labor *ad valorem* tax rates. When the production of the public goods is financed with a tax on capital, a lump sum tax, the optimal level of public good production is 2.99. On the other hand, when a tax on labor (a distorting tax) is used the optimal level is lower, 2.93. These results illustrate the tradeoff of the benefits of public good production against the costs of tax distortions. The optimal level of public good production is lower when a distorting tax is used to finance its production than when a non-distorting tax is used.

While this result appears to be intuitive, and arguably is typical (see Stiglitz, 2000, pp. 148-149), it need not be the case. For example, if the taxed good is a complement of the public good, the "effective" opportunity cost of producing the public good would be reduced below its "physical" opportunity cost because additional public good production stimulates tax revenue as a consequence of the complementarity (see Atkinson and Stiglitz, 1980, pp. 490-492). Naturally, the demand responsiveness of the taxed good to the tax further complicates the analysis. The ultimate effect on the optimal level of public good production is ambiguous; it is even possible for the optimal level to increase when production is financed by a distorting rather than lump sum tax (see Atkinson and Stern, pp. 123-126).

While our simulations only illustrate the more intuitive result (moving from a non-distorting to a distorting method of finance reduces the optimal level of the public good), they do reinforce the basic lesson general equilibrium analysis teaches: we cannot just look at one aspect of the economy in isolation. The standard public good optimization rule which only considers the benefits and costs of producing the public good (the sum of marginal rates of substitution equal the rate of product transformation) is not the end of the story. We must also account for the impact of the taxes raised to finance the production of the public good on other markets.

ILLUSTRATION 5: EXTERNALITITES AND EFFICIENCY

As a final illustration, we add an externality. An additional term is added to the household's utility function to allow the externality to affect the household directly (we continue to use a single household to abstract from distributional concerns):

$U = X^{.5}\,Y^{.3}\,L^{.2}\,G^{.1}\,C^{.2}$ where $C = 10 - E$ and E = External Effect[3]

The variable C can be thought of as clean air and E pollution. Originally, there are 10 units of clean air available for the household to enjoy, but the external effect pollutes the air reducing the quantity of clean air available to the household.

In this simulation, firm 2, is the polluter. Each unit of good Y produced results in 0.2 units of pollution:

Firm 1 Firm 2

[3] Although it is irrelevant for the simulations, C is actually defined so that it is nonnegative: $C = \max[10 - E, 0]$

$$X = L^{.8} K^{.2} \qquad Y = L^{.2} K^{.8}$$
$$E = .2Y$$

Table 6 reports on two scenarios: one in which labor is taxed, but the externality is not, and a second in which the labor is taxed and pollution is subject to a Pigovian tax of .1 per unit:[4] In the absence of the Pigovian tax, the optimal level of public good production is 2.93; in this case, the optimal tax rate on labor is 25 percent and 2.97 units of pollution are produced. When the Pigovian tax is imposed and the tax rate on labor remains at 25 percent, optimal public good production rises because more tax revenue is generated and the amount of pollution falls; both of these effects cause utility to increase from 19.342 to 19.375. Welfare can be improved even more, however, by reducing the distorting tax on labor. A reduction in the labor tax rate from 25 to 20 percent increases utility from 19.375 to 19.428. This occurs because the tax revenue generated by taxing the externality reduces the need to generate tax revenue from the distorting tax on labor. In the addendum to Table 6, we calculate the changes in utility arising from each of these two effects thereby illustrating the "double dividend" potentially available from environmental taxes. In this case then, the reduction in the labor tax distortion contributes substantially to the

[4] The externality tax is a unit tax of .10 per unit of the external effect when the prices are normalized to sum to 1. Note that an *ad valorem* tax on the external effect would not make sense since in the absence of government intervention, the "price" of the external effect is 0.

increase in utility. One reason that the double dividend is so large here is that the tax system originally favors good Y, the capital intensive good, which is also the good producing the pollution. In other situations, the double dividend might be smaller or even negative (see Salanie, 2003, pp. 200-204). Again, the general equilibrium models illustrate the importance of considering all of the ramifications of the fact that markets are interconnected.

CONCLUSION

The primary reason that students should study general equilibrium theory is to learn more about economics generally. As currently taught in intermediate microeconomics courses, general equilibrium theory yields precious few such insights. Other than developing an understanding of Pareto optimality and grasping a vague notion that "everything affects everything", students emerge from the typical course thinking that general equilibrium is probably the least important part of economic theory for understanding the practical world. We believe that nothing could be further from the truth. The impacts of most important economic policies can only be fully understood in a general equilibrium context. Strengths and limitations of current research in these areas can only be understood if someone is familiar with how actual general equilibrium models work. We believe that the general equilibrium approach described in this paper provides a relatively simple way for students to begin to develop such an understanding.

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Table 1: Coverage of General Equilibrium Concepts in Leading Microeconomics Texts Concept

	Browning/Zupan	Frank	Katz/Rosen	Landsburg	Nicholson/Snyder	Perloff	Pindyck/Rubinfeld	Varian
Reasons for General Equilibrium	Y	N	Y	Y	Y	Y	Y	N
Pareto Efficiency	Y	Y	Y	Y	Y	Y	Y	Y
Exchange/Edgeworth Box	Y	Y	Y	Y	Y	Y	Y	Y
Initial Endowments	Y	Y	Y	Y	Y	N	Y	Y
Production and Efficiency	Y	Y	Y	Y	Y	Y	Y	Y
Walras' Law	N	N	N	N	N	N	N	Y
Existence of Walrasian Equilibrium	N	N	N	N	N	N	N	Y
CGE Models	N	N	N	N	N	N	N	N
First Theorem	Y	Y	Y	Y	Y	Y	Y	Y
Market Failure	Y	Y	Y	N	Y	N	Y	Y
Second Theorem	N	N	Y	N	Y	N	N	Y
Social Welfare Function	N	N	Y	N	N	Y	N	Y
Detailed Applications of GE Models	N	N	N	N	Y	N	N	N
Comparisons of General versus Partial Models	N	N	N	N	N	N	N	N

Table 2: A Commodity Tax and Redistribution – Illustration 1

	Simulation 1 No Taxes		Simulation 2 $t_X = .4$	
Prices				
X	.3629		.4732	
Y	.2530		.2178	
L	.2481		.1880	
K	.1360		.1209	
Price Ratios				
P_X/P_Y	1.4344		2.1725	
P_L/P_K	1.8243		1.5554	
Production				
Firm	1	2	1	2
X	23.76		17.95	
Y		25.08		28. 84
L	–27.81	–5.11	–21.68	–6. 68
K	-12.68	–37.32	–8.43	–41.57
Taxes	.00	.00	3.40	.00
Consumption				
Household	1	2	1	2
Redist[5]			.50	.50
X	15.70	8.06	11.67	6.27
Y	13.51	11.57	15.22	13.63
L	9.19	5.90	11.75	7.89
K	.00	.00	.00	.00
Income	11.40	7.32	11.05	7.42
Transfers	.00	.00	1.70	1.70
Utility	13.48	8.75	12.66	8.96

[5] "Redist" represents the redistribution factors. In simulation 1, there is no reason to specify them; since no taxes present, there is no tax revenue to redistribute. In simulation 2, tax revenue is collected and the factors must sum to 1 to satisfy the government budget constraint.

Table 3: Commodity and Input Taxes Equivalence – Illustration 2

	Simulation 1 No Taxes		Simulation 3 $t_X = .20$ and $t_Y = .20$ $t_L = 0$ and $t_K = 0$		Simulation 4 $t_X = 0$ and $t_Y = 0$ $t_L = .25$ and $t_K = .25$	
Prices						
X	.3629		.3989		.3989	
Y	.2530		.2667		.2667	
L	.2481		.2213		.2213	
K	.1360		.1131		.1131	
Price Ratios						
P_X/P_Y	1.4344		1.4961		1.4961	
P_L/P_K	1.8243		1.9569		1.9569	
Production						
Firm	1	2	1	2	1	2
X	23.76		22.28		23.76	
Y		25.08		24.80		25.08
L	–27.81	–5.11	–25.71	–4.78	–27.81	–5.11
K	-12.68	–37.32	–12.58	–37.42	–12.68	–37.32
Taxes	.00	.00	1.78	1.32	.60	1.77
Consumption						
Household	1	2	1	2	1	2
Redist			.50	.50	.50	.50
X	15.70	8.06	14.27	8.01	14.27	8.01
Y	13.51	11.57	12.81	11.99	12.81	11.99
L	9.19	5.90	10.29	7.22	10.29	7.22
K	.00	.00	.00	.00	.00	.00
Income	11.40	7.32	11.39	7.99	11.39	7.99
Transfers	.00	.00	1.55	1.55	1.55	1.55
Utility	13.48	8.75	12.94	9.22	12.94	9.22

Table 4: Labor Tax versus Capital Tax – Illustration 3

	Simulation 1 No Taxes		Simulation 5 $t_L = 0$ $t_K = .5$		Simulation 6 $t_L = .5$ $t_K = 0$	
Prices						
X	.3629		.3801		.4098	
Y	.2530		.2650		.2614	
L	.2481		.2599		.1924	
K	.1360		.0950		.1364	
Price Ratios						
P_X/P_Y	1.4344		1.4344		1.5676	
P_L/P_K	1.8243		2.7364		1.4102	
Production						
Firm	1	2	1	2	1	2
X	23.76		23.76		21.18	
Y		25.08		25.08		24.32
L	–27.81	–5.11	–27.81	–5.11	–24.06	–4.41
K	-12.68	–37.32	–12.68	–37.32	–12.72	–37.28
Taxes	.00	.00	.60	1.77	2.31	.42
Consumption						
Household	1	2	1	2	1	2
Redist			.80	.20	.56	.44
X	15.70	8.06	15.70	8.06	14.16	7.02
Y	13.51	11.57	13.51	11.57	13.32	11.00
L	9.19	5.90	9.19	5.90	12.07	7.47
K	.00	.00	.00	.00	.00	.00
Income	11.40	7.32	11.94	7.66	11.61	7.19
Transfers	.00	.00	1.90	.47	1.53	1.20
Utility	13.48	8.75	13.48	8.75	13.47	8.50

Table 5: Financing Public Goods – Illustration 4

K or L Tax Rate	Ad Valorem K Tax Both Firms		Ad Valorem L Tax Both Firms	
	Public Good	Utility	Public Good	Utility
.10	1.02	12.594	1.35	12.805
.15	1.48	12.875	1.93	13.014
.20	1.90	13.022	2.45	13.088
.25	2.29	13.096	**2.93**	**13.094**
.30	2.65	13.127	3.37	13.060
.35	**2.99**	**13.130**	3.77	13.002
.40	3.31	13.114	4.15	12.928
.45	3.61	13.086	4.49	12.845
.50	3.90	13.048	4.82	12.755

Table 6: Externalities and Taxes – Illustration 5

L Tax Rate	Ad Val L Tax and No Externality Tax			Ad Val L Tax and Externality Unit Tax (τ_E=.10)		
	Externality	Public Good	Utility	Externality	Public Good	Utility
.10	3.19	1.35	18.796	2.94	2.24	19.332
.15	3.10	1.93	19.147	2.87	2.80	19.426
.20	3.03	2.45	19.297	**2.80**	**3.31**	**19.428**
.25	**2.97**	**2.93**	**19.342**	2.74	3.77	19.375
.30	2.91	3.37	19.324	2.69	4.20	19.289
.35	2.85	3.77	19.267	2.64	4.60	19.182
.40	2.80	4.15	19.184	2.59	4.96	19.060

	Effect of Externality Tax	Effect of Lower Labor Tax	Total Effect
Change in Utility	.033	.053	.086
Percent of Total Change	38.4%	61.6%	100.0%