Amherst College
Economics 58
Fall 2010

**Solutions to PS # 3**

**1. CES Utility**

a. $MRS = \dfrac{\partial U/\partial x}{\partial U/\partial y} = (x/y)^{\delta-1} = p_x/p_y$ for utility maximization.

Hence, $x/y = (p_x/p_y)^{1/(\delta-1)} = (p_x/p_y)^{-\sigma}$ where $\sigma = 1/(1-\delta)$.

b. If δ = 0, $x/y = p_y/p_x$ so $p_x x = p_y y$.

c. Part a shows $p_x x/p_y y = (p_x/p_y)^{1-\sigma}$

Hence, for $\sigma < 1$ the relative share of income devoted to good *x* is positively correlated with its relative price. This is a sign of low substitutability.

For $\sigma > 1$ the relative share of income devoted to good *x* is negatively correlated with its relative price – a sign of high substitutability.

d. The algebra is a bit tricky here, but worth doing once. Let's solve for indirect utility

$$\frac{x}{y} = \left(\frac{p_x}{p_y}\right)^{-\sigma} \quad or \quad x = y\left(\frac{p_x}{p_y}\right)^{-\sigma}$$

Substituting into the budget constraint yields

$$I = p_x y\left(\frac{p_x}{p_y}\right)^{-\sigma} + p_y y \quad or \quad y = \frac{Ip_y^{-\sigma}}{p_x^{1-\sigma} + p_y^{1-\sigma}}$$

Similarly,

$$x = \frac{Ip_x^{-\sigma}}{p_x^{1-\sigma} + p_y^{1-\sigma}}$$

Hence, $\delta U = x^\delta + y^\delta = I^\delta\left(\dfrac{p_x^{-\sigma}}{p_x^{1-\sigma} + p_y^{1-\sigma}}\right)^\delta + I^\delta\left(\dfrac{p_y^{-\sigma}}{p_x^{1-\sigma} + p_y^{1-\sigma}}\right)^\delta$

Now $-\delta\sigma = 1-\sigma$ so $\delta U = I^\delta\left(\dfrac{1}{(p_x^{1-\sigma} + p_y^{1-\sigma})^{\delta-1}}\right)$ or

$$V' = I(p_x^{1-\sigma} + p_y^{1-\sigma})^{-1/1-\sigma} \quad where \quad V' = (\delta U)^{1/\delta}$$

This is the indirect utility function. Clearly it is homogeneous of degree zero in income and prices. Inverting the expression yields the expenditure function:

$$E = I = V'(p_x^{1-\sigma} + p_y^{1-\sigma})^{1/(1-\sigma)}$$

Clearly this is homogeneous of degree one in the prices. Note that the odd form for *V'* here suggests the use of the CES form given in Problem 4.13 in applications involving these functions.

**2. CES Indirect Utility and Expenditure Functions**

a. See prior problem

b. Scale all variables by t and the function is unchanged.

c. The partial derivative of *V* w.r.t. *I* is positive as the prices are positive.

d. Again, partial derivatives of *V* w.r.t. the prices are both negative: for example, $\partial V / \partial p_x = -Ip_x^{r-1}(p_x^r + p_y^r)^{-(1+r)/r} < 0$.

e. Simply reversing the positions of *V* and *I* in the indirect utility function yields $E = V(p_x^r + p_y^r)^{1/r}$.

f. Multiplying prices by any factor *t* multiplies expenditures by *t*.

g. For example, $\partial E / \partial p_x = Vp_x^{r-1}(p_x^r + p_y^r)^{(1-r)/r} > 0$.

h. $\partial^2 E / \partial p_x^2 = (1-r) \cdot Vp_x^{2r-2} K^{(1-2r)/r} + (r-1)Vp_x^{r-2} K^{(1-r)/r}$ where $K = (p_x^r + p_y^r)$. Division of this expression by $Vp_x^{r-2} K^{(1-r)/r}$ yields $(r-1)(1-k) < 0$ where $k = p_x^r / K < 1$.

**3.** a. Because of the fixed proportions between *h* and *c*, we know that the demand for ham is $h = I/(p_h + p_c)$. Hence

$$e_{h,p_h} = \frac{\partial h}{\partial p_h} \cdot \frac{p_h}{h} = \frac{-I}{(p_h + p_c)^2} \cdot \frac{p_h(p_h + p_c)}{I} = \frac{-p_h}{(p_h + p_c)}.$$

Similar algebra shows that $e_{h,p_c} = \dfrac{-p_c}{(p_h + p_c)}$. So, if

$p_h = p_c, e_{h,p_h} = e_{h,p_c} = -0.5$.

b. With fixed proportions there are no substitution effects. Here the compensated price elasticities are zero, so the Slutsky equation shows that $e_{x,p_x} = 0 - s_x = -0.5$.

c. With $p_h = 2p_c$ part a shows that $e_{h,p_h} = \frac{-2}{3}, e_{h,p_c} = \frac{-1}{3}$.

d. If this person consumes only ham and cheese sandwiches, the price elasticity of demand for those must be -1. Price elasticity for the components reflects the proportional effect of a change in the price of the component on the price the whole sandwich. In part a, for example, a ten percent increase in the price of ham will increase the price of a sandwich by 5 percent and that will cause quantity demanded to fall by 5 percent.

**4.** Hausman's terminology refers to the terms in the Taylor expansion of the expenditure function. For the introduction of new goods, price falls from $p^*$ to $p^1$ and the reduction in necessary expenditure is represented by the first order term in the Taylor expansion (notice that this happens because the envelope theorem shows that $\frac{\partial E}{\partial p_x} = x^c$ (which is denoted by $h_u$ in the paper. Substitution bias occurs because people change what they buy in response to changing prices. Such reactions to changing prices are captured by the second order term in the Taylor expansion – that is by the term in $\frac{\partial x^c}{\partial p_x} = \frac{\partial^2 E}{\partial p_x^2}$.

Consumer Surplus is given by the area under the Hicksian demand curve in Figure 1. When price falls from $p^2$ to $p^1$, the total gain in consumer surplus is given by the rectangular area plus the shaded triangle. The gain in Consumer surplus from the substitution effect is given only by the area of the shaded triangle. Hence, Hausman argues, focusing on substitution bias misses a lot – especially when it comes to new goods.