Amherst College
Department of Economics
Economics 58
Fall 2006

Name_Solutions_______

Second Examination

(Theory of the Firm)

There are three questions on this fifty minute examination. Each question is worth 20 points.

1. Many functions encountered in the theory of the firm can be expressed most easily through logarithms. This problem will explore a few simple examples using the logarithmic production function $\ln q = 0.5 \ln k + 0.5 \ln l$.

a. Show that cost-minimization requires that $wl = vk$ (Hint: this works easiest if you minimize the cost of producing a particular value of $\ln q$ rather than q itself).

$$\Im = vk + wk + \lambda(\ln q - 0.5\ln k - 0.5\ln l)$$

$$\frac{\partial \Im}{\partial k} = v - \frac{0.5\lambda}{k} = 0$$

$$\frac{\partial \Im}{\partial l} = w - \frac{0.5\lambda}{l} = 0$$

Hence: $\frac{v}{w} = \frac{l}{k}$ or $wl = vk$

b. Use your results from part a. to show that the total cost function in this case can be written $\ln C = \ln 2 + \ln q + 0.5 \ln v + 0.5 \ln w.$ (Hint: Use $C = vk + wl = 2vk = 2wl$ and take logs).

Using the hint:

$$\ln C = \ln 2 + \ln v + \ln k = \ln 2 + \ln w + \ln l \quad \text{so}$$

$$2\ln C = 2\ln 2 + \ln v + \ln w + \ln k + \ln l = 2\ln 2 + \ln v + \ln w + 2\ln q$$

$$\ln C = \ln 2 + \ln q + 0.5\ln v + 0.5\ln w$$

c. Use the cost function in part b to discuss what the average and marginal cost curves look like for this firm.

The production function is Constant Returns $q = k^{0.5} l^{0.5}$ one might suspect that *AC=MC* for all values of *q*. One way to show this is:

$$\frac{\partial \ln C}{\partial \ln q} = 1 = \frac{\partial C}{\partial q} \cdot \frac{q}{C} \qquad \text{Hence: } \frac{C}{q} = AC = \frac{\partial C}{\partial q} = MC \quad \text{regardless of } q.$$

d. True cost functions must exhibit a certain degree of homogeneity. State the homogeneity property that must hold for all cost functions and show that it applies in this particular case.

Cost functions are homogeneous of degree 1 in the input prices. Here

$$\ln C' = \ln 2 + \ln q + 0.5 \ln tv + 0.5 \ln tw = \ln 2 + \ln t + \ln q + 0.5 \ln v + 0.5 \ln w = \ln(tC)$$

e. For any cost function it is the case that $\frac{\partial \ln C}{\partial \ln w} = s_l$ where $s_l = \frac{wl}{C}$ is the share of labor costs in total costs. Prove this general result and then use the cost function in its logarithmic form to calculate s_l in this case. Describe this result intuitively.

In general, $\frac{\partial \ln C}{\partial \ln w} = \frac{\partial C}{\partial w} \cdot \frac{w}{C} = \frac{wl}{C} = s_l$ by Shephard's Lemma.

In this case $\frac{\partial \ln C}{\partial \ln w} = 0.5$. So labor's share is 50 percent – as suggested by the Cobb-Douglas exponents in part a.

2. One of the major reasons to undertake an extensive study of the theory of the firm is to understand input demand better. This question asks you to explore a firm's demand for labor in the simplest possible situation.

a. Suppose a firm's production function is given by $q = l^{0.5}$. Calculate the firm's total cost function and use that function to show that $\frac{\partial l^c}{\partial w} = \frac{\partial^2 C}{\partial w^2} = 0$.

$$C = wl = wq^2 \qquad l^c = \frac{\partial C}{\partial w} = q^2 \qquad \frac{\partial l^c}{\partial w} = 0$$

b. Explain the result in part a intuitively – what does it mean and why does it occur.

Because there is only one input here, there is no substitution effect when *w* changes. Producing a given *q* requires a certain amount of *l*, so if *q* is constant, so is *l*.

c. Calculate the profit function in this situation. Use it to show that $\frac{\partial l}{\partial w} = -\frac{\partial^2 \Pi}{\partial w^2} < 0$.

For profit maximization, produce where $P = MC = 2wq \quad q = \frac{P}{2w}$.

Hence

$$\Pi(P, w) = Pq - C = \frac{P^2}{2w} - \frac{P^2}{4w} = \frac{P^2}{4w} \quad l = -\frac{\partial \Pi}{\partial w} = \frac{P^2}{4w^2}$$

$$\frac{\partial^2 \Pi}{\partial w^2} = \frac{-P^2}{2w^3} < 0$$

d. Explain your results from part c intuitively – what does the result mean and why is the result here different from that in parts a and b?

Although there is no substitution effect from an increase in *w* in this problem, there is a negative output effect – an increase in *w* raises marginal cost and causes the firm to produce less, thereby hiring less labor.

e. More generally, what does this problem tell you about the firm's demand for labor input? Please be as precise as possible.

The change in the demand for an input in response to a change in the price of that input induces both negative substitution effects and negative output effects. Even if there are no substitution effects, the demand curve for an input will still be downward sloping because of output effects.

3. Thompson and Taylor's paper "The Capital-Energy Substitutability Debate: A New Look" seeks to summarize a number of prior studies of the relationship between the firm's use of capital and energy. You are to address the following questions about this piece.

a. What is the economic importance of this article? That is, what facts about the economy does this paper shed light on?

The article is concerned with how firms respond to changes in energy prices. The particular fear is that if energy and capital are complements, an increase in the price of energy might reduce firms' demands for capital and this might slow the rate of economic growth.

b. Why do the authors prefer to report elasticities of substitution between capital and energy based on the Morishima definition rather than on the Allen definition? What advantage(s) do the former elasticities provide?

The main reason that the authors prefer the Morishima definition is that it more accurately reflects movements along a given isoquant (with *q* held constant) in response to changing input prices. There is no requirement that inputs other than *e* and *k* be held constant in this definition – only that *q* remain constant as firms vary their use of *e* and *k*. The authors also give some practical reasons for preferring the Morishima definition – primarily that it is not so subject to erratic estimates in econometric work.

c. The authors claim that the Allen definition of substitutability is symmetric whereas the Morishima definition is not. How do they know that?

The authors give the following definition of the Allen measure:

$AES_{ij} = \frac{CC_{ij}}{C_i C_j}$ This definition is clearly symmetric – Notice also that

$$AES_{ij} = \frac{\partial x_i / \partial w_j}{\frac{x_i x_j}{C}} = \frac{\frac{\partial x_i}{\partial w_j} \cdot \frac{w_j}{x_i}}{\frac{w_j x_j}{C}} = \frac{e_{ij}^c}{s_j}$$ as claimed in the text.

$MES_{ji} = \frac{\partial \ln(x_j / x_i)}{\partial \ln(w_i / w_j)}$ as given in T&T and in N. Now if the price of input *i* changes, we have: $MES_{ji} = \partial \ln x_j / \partial \ln w_i - \partial \ln x_i / \partial \ln w_i = e_{ij}^c - e_{ii}^c$.

If we were to change the price of the *j*th input we would get $MES_{ij} = e_{ij}^c - e_{jj}^c$. These are not symmetric because $e_{ii}^c \neq e_{jj}^c$.

d. The authors draw an important conclusion from the asymmetry of the Morishima definition. What is it?

The authors show that $MES_{ke} > MES_{ek}$ and therefore conclude that increases in energy prices (say through taxes) will have a larger effect on capital energy substitution than would subsidies to energy-saving capital.