Testing Conservation of Energy

In class we demonstrated that if gravity is the only force doing work on an object that the total mechanical energy is conserved. That is

$$E = K + U = \tfrac{1}{2}mv^2 + mgh$$

should be constant for the motion. We would like to test this. Notice that if it is true, then

$$2E/m = v^2 + 2gh$$

Or rearranging the equation

$$v^2 = 2E/m - 2gh \qquad \text{(Equation 1)}$$

Since E, m and g are all constants, if conservation of energy holds, then if we were to make a plot with v^2 on the y-axis and h on the x-axis, we would expect to get a straight line with a slope of -2g and an intercept of 2E/m. If conservation of energy does not hold, then this plot will not yield a straight line.

The experiment

Obviously, in order to test this, we will need to be able to measure both the velocity and the height of an object as it moves under the influence of gravity. The apparatus for doing this is described here.

Apparatus:

Timer module (ME-9283), paper tape, carbon paper disks, 200-g mass and clip, support rod and clamps, ladder, masking tape, meter stick.

Experiment:

In order to investigate this process, we must measure the position of a falling object as a function of time. To do this, you will have a weight that can be attached to one end of a piece of paper tape. The tape passes through a small timing module (see figures 1 and 2). The module contains a small oscillating hammer (not visible) that 40 times/sec strikes a sandwich made up of a piece of carbon paper, the tape, and a strike plate. As the tape passes through the module, a series of dots is recorded on the tape. From these dots, you can determine how the position of the falling mass varies with time.

1) Set up the apparatus as shown in Figure 2. The tape should be ~1.5 m long, and the bucket of sand should be positioned so that the falling weight will hit it rather than the floor. Hold the tape *vertically* above the timer module and turn on the module to 40 Hz (40 cycles/sec). Make certain that the mass is not swinging. Then release the tape. The dots on the tape now should have a recording of the position of the mass as a function of time.

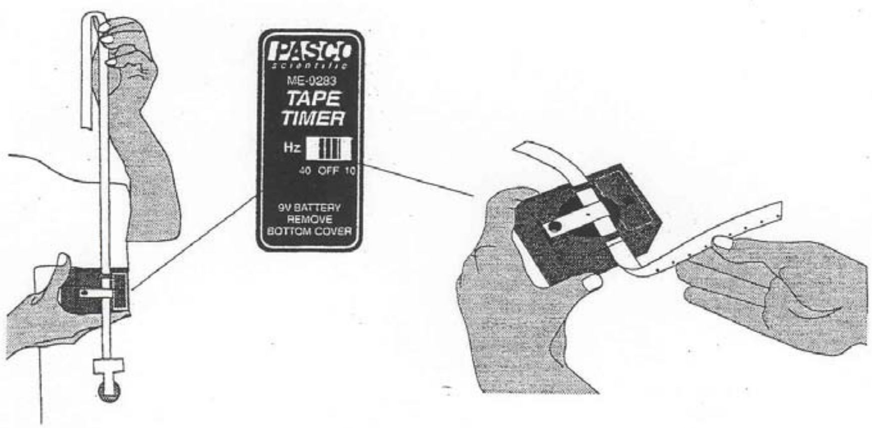

Figure 1

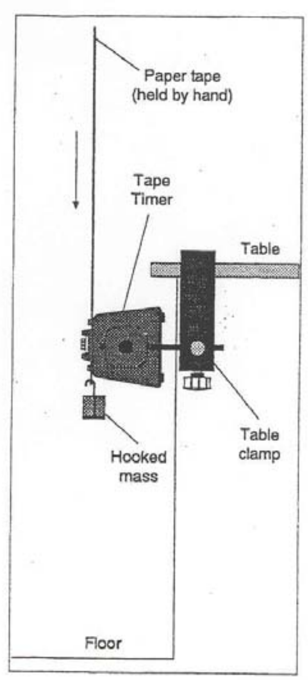

Figure 2

2) Choose a well-defined dot just beyond the confusion of dots at the beginning of the tape. Label this first good dot on your strip 0 and label successive points 1, 2, 3, etc. until you get to the last of the recorded points. The time at which each of these points was created is this label number multiplied by (1/40) s (the time between dots). We have arbitrarily chosen to call t = 0 the time at which the first clear dot was created.

3) Take and draw a reference line at the location of the last point on your tape. Lets call this point h = 0. (We can choose the origin of our coordinate system wherever we like. In this case we are calling h = 0 the height of our object when this last point was recorded). Measure the distance (in meters) of all of your other points from this reference point. Think about how to position the meter stick to avoid "parallax" problems. Record a table of values of h vs. t. It should look something like this (your numbers will of course be somewhat different).

Point	t(s)	h(m)
0	0	1.157
1	0.025	1.146
2	0.05	1.126
3	0.075	1.101
4	0.1	1.069
5	0.125	1.032
6	0.15	0.990
7	0.175	0.940
8	0.2	0.885
9	0.225	0.823
10	0.25	0.756
11	0.275	0.682
12	0.3	0.603
13	0.325	0.517
14	0.35	0.426
15	0.375	0.328
16	0.4	0.225
17	0.425	0.116
18	0.45	0.000

Analysis

In order to test conservation of energy, we need to extract the velocity of our mass from the height and time we have measured. We do this by using the expression

$$v = \Delta h / \Delta t$$

For any point (except the first and the last) we can get the velocity at this time by taking the differences between the preceding and following points. For example, if we wish to determine the velocity at point number 1, we take the difference between the objects height at point number 2 and point 0 to get Δh. The time change, Δt between these two points is 2 x (1/40)s = (1/20) s. This allows us to deduce the velocity at point 1.

Repeating this operation for all of the points, your table should now look something like this.

Point	t(s)	h(m)	v(m/s)	v^2(m/s)^2
0	0	1.157		
1	0.025	1.146	0.62	0.38
2	0.05	1.126	0.90	0.81
3	0.075	1.101	1.14	1.30
4	0.1	1.069	1.38	1.90
5	0.125	1.032	1.58	2.50
6	0.15	0.990	1.84	3.39
7	0.175	0.940	2.10	4.41
8	0.2	0.885	2.34	5.48
9	0.225	0.823	2.58	6.66
10	0.25	0.756	2.82	7.95
11	0.275	0.682	3.06	9.36
12	0.3	0.603	3.30	10.89
13	0.325	0.517	3.54	12.53
14	0.35	0.426	3.78	14.29
15	0.375	0.328	4.02	16.16
16	0.4	0.225	4.24	17.98
17	0.425	0.116	4.50	20.25
18	0.45	0.000		

In the table above, I have also added a column with the square of the velocity. We need this column in order to plot v^2 versus h. Add a column to your table that includes v^2.

Now you are ready to test conservation of energy and equation 1. Make a plot of v^2 versus h. Does your plot yield a straight line? If it does not, there is either a problem with your data or with the law of conservation of energy.

If you did get a straight line, determine its slope (be sure to keep the units). Equation 1 tells us that if we divide this slope by 2, we should obtain g, the acceleration of gravity. What value do you deduce from your data for g?

Equation 1 also tells us that the intercept of our plot should be $2E/m$. So the energy of our system is (m/2) x intercept. What is the energy of our system? (again, be careful with units.)

If we had chosen a different point to call h = 0, what would change in our analysis? Would we have gotten the same value of g? Would we have gotten the same value of E? Would conservation of energy still hold?

Note: Be sure to attach your tape to your lab write up.