**Math 211, Multivariable Calculus, Fall 2011**
**Midterm III Practice Exam 2**

You will have 50 minutes for the exam and are not allowed to use books, notes or calculators. Each question is worth 10 points.

1. (a) What does it mean to say that $(a, b)$ is a **saddle point** of the function $f(x, y)$?
   (b) Find the critical points of the function
   $$f(x, y) = x^3 - xy + y^2.$$
   (c) For each critical point, decide if it is a local maximum, local minimum or saddle point.

2. Find the absolute maximum of the function
   $$f(x, y) = x - y^2$$
   on the region
   $$x^2 + y^2 \leq 1.$$
   (Make sure you explain how you know that your answer is the absolute maximum.)

3. Let $R$ be the triangular region with vertices $(0, 0)$, $(0, 1)$, $(2, 2)$. Calculate
   $$\iint_R xy \, dA.$$

4. Let $D$ be the solid cylinder whose ends are given by the planes $z = -1$ and $z = 2$, and whose curved surface is given by $x^2 + y^2 = 1$. Calculate
   $$\iiint_D z \, dV.$$

5. The *hyperbolic coordinates* of a point in the $xy$-plane are the variables $s, t$ given (when $x > 0$) by
   $$s = xy, \quad t = \frac{y}{x}.$$
   (a) Find the Jacobian $\frac{\partial(x,y)}{\partial(s,t)}$ for the change of variables from $x, y$ to $s, t$.
   (b) Let $R$ be region in the $xy$-plane bounded by the lines $y = x$ and $y = 2x$, and the curves $y = \frac{1}{x}$ and $y = \frac{2}{x}$. Sketch a diagram of the region $R$.
   (c) Use hyperbolic coordinates to calculate the integral
   $$\iint_R xy \, dA.$$