

Math 13: Project 1

Due Friday 3/5

Introduction

With the introduction of a third dimension we are able to examine more complicated objects than just points and lines. In fact, you could find equations that model just about any object you can see. In this project you will be investigating some particular classes of surfaces that have fairly simple equations and occur frequently. It will be your task to understand the differences between these surfaces and model an object from the Mead Art Museum that can be described by these surfaces. The art museum has kindly agreed to aid us in working with the art and will be present to assist us. **Under no circumstances are you to touch any of the works at the museum.**

You will work in groups of 3-4 on this project and turn in a joint report. I expect that all members of the group will understand everything in the report and do an equal share of the work.

1. **Friday February 12:** We will go meet in the Mead Art Museum at 10am to obtain data for the surfaces. The museum has arranged for several staff members to take the actual measurements that you request. Remember that bags are not allowed in the museum and must be checked at the door, so allow ample time to get through this process. We will meet in the lobby of the museum.
2. I will block some time in Computer Lab 014 (of Seeley Mudd) to help with Mathematica. The time is TBD.
3. **Wednesday March 10, 7pm** Extra Credit at the Mead (see below).

Goals

1. Be able to distinguish the surfaces covered in 13.6 of Stewart by both form and equation.
2. Identify where the surfaces occur in everyday life.
3. Understand how varying the parameters of the equations affects the shape of the surfaces.
4. Be able to have Mathematica graph a given surface.

Assignment

1. Familiarize yourself with examples of spheres, cylindrical surfaces, quadric surfaces, ellipsoids, paraboloids, elliptic cones, and hyperboloids either from Stewart or from examples of your own creation.
2. Find an object in Mead Art Museum that can be modeled by some combination of these surfaces.
Be creative in finding your object. Consider sculptures and crafted works, consider objects within paintings, consider the architecture of the building itself, etc.
3. Write an equation that models the object as a surface (or collection of surfaces) and produce a computer aided drawing of your object.

4. You turn in a report containing at least
 - (a) description of your object with a brief history
 - (b) mathematical justification of your model
 - (c) your final mathematica rendering
 - (d) an image of the original object
 - (e) a diagram with your measurements.

and you should submit your Mathematica file electronically.

Extra Credit: For the Mead Art Museum The Mead Art Museum has invited the class to participate in a special exhibition in which your Mathematica diagrams and thoughts about the project will be shared alongside the artworks in the form of wall texts and a museum website feature.

The museum project will conclude with an extra-credit event: Pamela Russell from the museum would like to interview you briefly afterwards to record (on audiotape) your informal reminiscences of the project—why you selected the artworks you did; if you saw the artworks differently after the project than at the time you selected them; if your experiences with Mathematica have effected the way in which you see other things, etc. Pizza will be served at this optional, 1-hour event. **You will receive 3 points of extra credit to your final project grade for participating in this event.**

Rules for working with the art

We will be hosted by the art museum staff on during regular class time, where you will be assisted in learning about the art and obtaining data. **Under no circumstance are you to touch any of the works at the museum.** To prevent accidental damage, a good rule is to come no closer than 6" to any of the works. The staff will be glad to help you obtain your data. Be sure to bring a pencil as pens are not permitted in the museum.

Checklist for Your Writing Projects

Based on checklists by Annalisa Crannell at Franklin & Marshall and Tommy Ratliff at Wheaton College.

Does this paper:

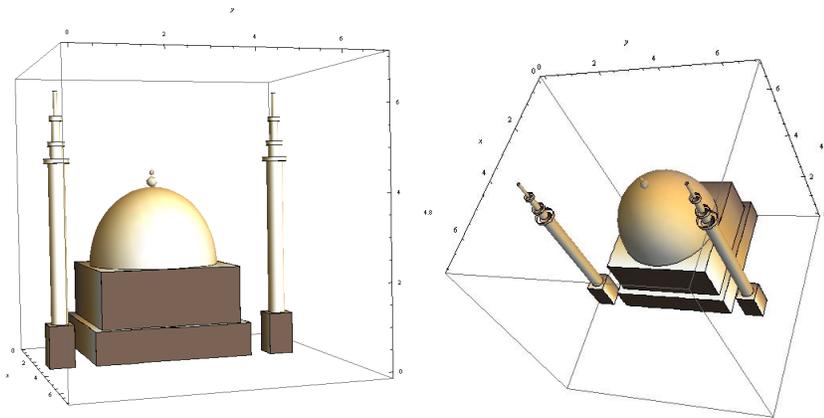
1. clearly (re)state the problem to be solved?
2. provide an explanation as to how the problem will be approached?
3. state the answer in a few complete sentences which stand on their own?
4. give a precise and well-organized explanation of how the answer was found?
5. clearly label diagrams, tables, graphs, or other visual representations of the math?
6. define all variables, terminology, and notation used?
7. clearly state the assumptions which underlie the formulas and theorems, and explain how each formula or theorem is derived, or where it can be found?
8. give acknowledgment where it is due?
9. use correct spelling, grammar, and punctuation?
10. contain correct mathematics?
11. solve the questions that were originally asked?

Grading

This project is worth 5% of your final grade and will be graded out of 25 points with the following criteria.

- 10 point for discussion of the object and method.
- 10 points for justification of the model.
- 5 points for final rendering of the model.

An example rendering from a previous class



Introduction to Mathematica

Mathematica is a computer program designed to aid in doing mathematics. In this project you mainly need a knowledge of how to use its built in functions. Mathematica knows about most of the most common mathematical functions such as $\sin(x)$, $\cos(x)$, $\ln(x)$, \sqrt{x} , etc.. When using Mathematica's built-in functions there 2 very important things to remember:

1. All built-in functions have the first letter CAPITALIZED.
2. The inputs to functions are placed inside SQUARE BRACKETS.

So, for example, if you want to know the square root of 19, you can type

$$\text{Sqrt}[19] \tag{1}$$

Built-in Constants

Mathematica also has number of built-in constants such as π (Pi) and e (E). It also knows about infinity (Infinity) and the complex number i (I). Notice that the built-in constants also have the first letter capitalized.

Arithmetic Operations

The arithmetic operations in Mathematica are probably similar to what you have used on a graphing calculator:

1. +, -, *, / are used for addition, subtraction, multiplication and division.
2. Round parentheses are used for grouping.
3. The caret symbol is used for exponents (x^2).
4. Parenthesis and the multiplication symbol * can be omitted in the same situations were we omit them in mathematics, but it never hurts to use them anyway.

Examples

Try these examples. Note that you should see a horizontal line across the notebook before you begin (if not, click on the notebook) and that you must type ENTER (not return) to get a command to execute (Shift-return also works).

| <u>Math</u> | <u>Mathematica</u> |
|-------------------|-----------------------------|
| $\sqrt{4 + 5\pi}$ | <code>Sqrt[4 + 5 Pi]</code> |
| $\sin(\pi/3)$ | <code>Sin[Pi / 3]</code> |
| $\ln(e^2 + e)$ | <code>Log[E^2 + E]</code> |

Some Special Functions

`N[]`

When it is able to Mathematica will generally give exact answers rather than approximate answers, i.e. $\frac{1}{3}$ instead of .33333. If you want a decimal approximation and Mathematica has given you an exact value, you can use the built-in function `N[]` to request an exact value. For example, to get an approximation for $\frac{55}{34}$, enter `N[55/34]`.

If you get an exact value and want a decimal approximation, you can use a short-cut: `N[%]` will give the numerical approximation to the last output. Note that `%` always stands for the last output in Mathematica.

`?`

The question mark can be used to get information about functions. For example

$$?N \quad (2)$$

will tell you something about the function `N[]`.

$$??N \quad (3)$$

will give somewhat different information. And

$$?N* \quad (4)$$

will give you a list of Mathematica functions that begin with the letter N. If none of these methods give you the information you need, you can also look in the extensive help menus.

`Expand[]`

Notice that if you enter $(x + 1)^2$ Mathematica returns $(x + 1)^2$ to save space. You can ask Mathematica to expand such expression by entering

$$\text{Expand}[(x + 1)^2]. \quad (5)$$

Defining your own functions

In mathematics we often give names to objects to make it easier to refer to them. For example

$$f(x) = x^2 + 1 \quad (6)$$

means the we are defining the function f to take a input value x to the value $x^2 + 1$. Mathematica can also do this. For example

$$f[x_] := x^2 + 1 \quad (7)$$

defined the function f to take a single input and return its square plus 1. Notice the `_` after the variable x on the left hand side in the function definition and the colon before the equals sign. This tells Mathematica that x is a place holder for some value not yet entered and you are defining the function f . Having defined f we can now do things like

1. `f[1]` which evaluates the function f at 1.
2. `?f` which will return the definition of f .
3. `Clear[f]` which will remove the definition on the function f .
4. `Expand[f[x + 1]]`.
5. `Plot[f[x], {x, -2, 2}]` (Plot is defined below).

Here are some guidelines for defining your own functions

1. Always clear the function before defining it. This will avoid bad interactions with previous definitions.
2. An underscore must follow the input variable.
3. Usually you will want to `:=` to define a function. There are other kinds of equal signs but don't use another kind of equals sign unless you are sure you know what that would mean.
4. You don't need to use one letter names, but it is a good idea to NOT capitalize your functions to avoid conflicts with built-in Mathematica functions.
5. After defining a function is a good idea to try `?` to check the definition and enter a test value to check the evaluation.

Graphing

We introduce two of the most basic plotting functions here. You can find many more in the Visualization and Graphics section of the help (start with Function Visualization).

Plot[]

The Plot function returns the graph of a function. For example:

$$\text{Plot}[\text{Sin}[x], \{x, -2 * \text{Pi}, 2 * \text{Pi}\}] \quad (8)$$

This tells Mathematica to plot the function $\sin x$ where x goes from -2π to 2π .

You can also use plot to put several functions on the same graph, such as with

$$\text{Plot}[\{f(x), g(x), h(x)\}, \{x, -2, 2\}] \quad (9)$$

There are a number of other options to the Plot[] command controlling things like colors, thickness of lines, aspect ratio, range of output values to display, etc. Consult ?Plot for more information.

Plot3D[]

The Plot3D function is used to make 3-dimensional graphs. For example

$$\text{Plot3D}[\text{Sqrt}[1 - x^2 - y^2], \{x, -1, 1\}, \{y, -1, 1\}] \quad (10)$$

will draw the upper hemisphere of a sphere of radius 1. Notice the the function is of the form

$$z = f(x, y) \quad (11)$$

Note that Plot3D can also take a list of functions as input. For example, we can intersect the plane $z = .5$ with the upper hemisphere of the unit sphere with

$$\text{Plot3D}[\{\text{Sqrt}[1 - x^2 - y^2], .5\}, \{x, -1, 1\}, \{y, -1, 1\}] \quad (12)$$

ContourPlot3D[]

The function allows you to plot an implicit function (i.e. you do not need to solve for z . For example you can plot a sphere with

$$\text{ContourPlot3D}[x^2 + y^2 + z^2 == 1, \{x, -1, 1\}, \{y, -1, 1\}, \{z, -1, 1\}]$$

You can also plot a list of functions by enclosing them with $\{ \}$ as in the other functions.

Show[]

If you break up your object into individual pieces you can name each by entering

$$\text{piece1} := \text{ContourPlot3D}[x^2 + y^2 + z^2 == 1, \{x, -1, 1\}, \{y, -1, 1\}, \{z, -1, 1\}]$$

where “piece1” is whatever name you wish to name this part of your object. You can then combine all of the pieces into a single plot with the Show function.

$$\text{Show}[\text{piece1}, \text{piece2}, \text{piece3}]$$

You can have as many named pieces as you want in the show function.

RegionPlot3D[]

This allows you to plot an inequality. For example

$$\text{RegionPlot3D}[z^2 >= x^2 + y^2, \{x, -1, 1\}, \{y, -1, 1\}, \{z, -1, 1\}]$$

draw the cone with its interior filled in.

Some Plotting Options

Mesh You can specify the grid on the object with the Mesh function. For example

$$\text{ContourPlot3D}[x^2 + y^2 + z^2 == 1, \{x, -1, 1\}, \{y, -1, 1\}, \{z, -1, 1\}, \text{Mesh} \rightarrow \text{None}]$$

AxesLabel Can provide a label for you axes.

ContourPlot3D[$x^2 + y^2 + z^2 == 1$, { x , -1, 1}, { y , -1, 1}, { z , -1, 1}, *AxesLabel* → {" x'' ", " y'' ", " z'' "}]

PlotStyle Allows you to specify colors for the Plot functions with

Plot3D[*Sqrt*[$1 - x^2 - y^2$], { x , -1, 1}, { y , -1, 1}, *PlotStyle* → *Blue*]

draws the function with blue.

ContourStyle Allows you to specify colors for the Contour functions with

ContourPlot3D[$x^2 + y^2 + z^2 == 1$, { x , -1, 1}, { y , -1, 1}, { z , -1, 1}, *ContourStyle* → *Blue*]

PlotRange Allow you to set the maximum x , y , and z values for the axes to contain. To avoid distortion you should have the same range for each axes. (i.e. $-10 \leq x \leq 10$, $-10 \leq y \leq 10$, $0 \leq z \leq 20$ would be fine, since they all span 20 units.)

Show[*P1*, *P2*, *P3*, *PlotRange* → {{-10, 10}, {-10, 10}, {0, 20}}}]

Colors You can set *PlotStyle* or *ContourStyle* to have colors. There are some built in colors in Mathematica like "Blue", or you can use *RGBColor* to specify the red, green, blue values of the color you want. Note that the ranges for the RGB is [0, 1].

ContourPlot3D[$x^2 + y^2 + z^2 == 1$, { x , -1, 1}, { y , -1, 1}, { z , -1, 1}, *ContourStyle* → *RGBColor*[.25, .33, .1]]

Opacity You can make your objects semi-transparent by setting the "Opacity" from 0 to 1.

ContourPlot3D[$x^2 + y^2 + z^2 == 1$, { x , -1, 1}, { y , -1, 1}, { z , -1, 1}, *ContourStyle* → *Opacity*[.5]]

Thickness You can set the thickness of the lines/surfaces by setting "Thickness".

ContourPlot3D[$x^2 + y^2 == 1$, { x , -1, 1}, { y , -1, 1}, { z , -1, 1}, *ContourStyle* → *Thickness*[.1]]

Graphing Disks and other inequalities You can graph an inequality for any of your surfaces using the *RegionPlot3D* function. The following makes a disk

RegionPlot3D[$x^2 + y^2 <= 1$, { x , -1, 1}, { y , -1, 1}, { z , 0, 0.001}, *PlotRange* → {{-1, 1}, {-1, 1}, {-1, 1}}]

Rotation If you have a surface that is rotated in some fashion it is quite possible to do this with Mathematica. You can either ask me or look up "RotationTransform" in the help.