Math 13: Project 2
Due Friday 11/20

1 Introduction

The Great Northern Paper Company in Millinocket, Maine, produces newsprint, computer paper, and many other kings of paper goods. In order to ensure an adequate supply of affordable power, it also operates six hydroelectric gathering stations on the Penobscot river. In the present problem we are concerned with the power station on the west branch of the Penobscot river, which gets its water from a dam on Ripogenus lake. A pipe sixteen feet in diameter and three-quarters of a mile long carries water from the dam to the power station, through an elevation drop of 170feet. The rate at which water flows through the pipe varies, depending on conditions in the watershed. Once at the power station, manually controlled valves and gates distribute the water to the station's three hydro-electric turbines. These turbines have known, and different "power curves," which give the amount of electric power generated as a function of the water flow sent to the turbine.

Remember that you should fully justify all of your work and your only sources of aid should be your book, your other group members, and me. This project will be graded out of 25 points.

2 Background

Commercial electricity id produced by turbines which turn mechanical energy into electric current. In some cases, coal, oil, gas, or atomic fuel is used to make steam which runs the turbines. Hydroelectric power stations use the energy of falling water to turn the turbines. The energy comes both from the weight of the water and from the "head" on it, that is the vertical distance through which the water falls. The basic equation which relates water flow to energy production was published by Daniel Bernoulii in 1738, and is called *Bernoulli's equation.* It results from applying the principle of conservation of energy to the flow between the lake surface and the turbine. In our context, the equations states that

$$W = \gamma Q_\eta (Z_h - Z_t - f), \tag{1}$$

where

$W =$ power extracted byu the turbine (foot-pounds/second)

$\gamma =$ specific weight of water (pounds/foot^3)

$Q =$ flow rate of fluid (feet^3/seconds, abbreviated cfs)

$\eta =$ turbine efficiency, a function of Q

$Z_h =$ elevation of the lake surface (feet)

$Z_f =$ elevation of the turbine (feet)

$f =$ energy loss doe to friction, a function of Q.

In our case the difference between Z_h and Z_t is 170 feet. The main factor in f is the energy lost as the water flows through the pipe. Engineers derived from experiment the estimate

$$f = 1.6 \cdot 10^{-6} Q_T^2 \tag{2}$$

where Q_T is the total water flow in cubic feet per second (cfs).

The efficiency η, which is a function of Q, differs for the three turbines. Experimental results suggested expressing γQ_η as a quadratic polynomial on Q, for each turbine. Statistical curve fitting then gave the following equations for the power output of the three turbines

$$KW_1 = (-18.89 + 0.1277Q_1 - 0.408 \cdot 10^{-5} Q_1^2)(170 - 1.6 \cdot 10^{-6} Q_T^2) \quad 250 \le Q_1 \le 1110 \tag{3}$$

$$KW_2 = (-24.51 + 0.1358Q_2 - 0.469 \cdot 10^{-5} Q_2^2)(170 - 1.6 \cdot 10^{-6} Q_T^2) \quad 250 \le Q_2 \le 1110 \tag{4}$$

$$KW_3 = (-27.02 + 0.1380Q_3 - 0.384 \cdot 10^{-5} Q_3^2)(170 - 1.6 \cdot 10^{-6} Q_T^2) \quad 250 \le Q_3 \le 1225 \tag{5}$$

where

Q_i = flow through the turbine i (cfs)

KW_i = power generated by turbine i (kilowatts)

Q_t = total flow through the station (cfs).

The coefficients in the quadratic polynomials include a scaling factor to transform units of mechanical power into units of kilowatts. The bounds on the Q_i's represents the fact that the turbines cannot operate with a flow below 250 cfs, or above a maximum flow which is slightly higher for turbine 3 than for turbines 1 and 2.

3 Problem

The power plant supervisor wants you to devise a plan for distributing water among the turbines which will get the maximum energy production from the three turbines for any (allowable) rate of water flow.

4 Checklist for Your Writing Projects

Based on checklists by Annalisa Crannell at Franklin & Marshall and Tommy Ratliff at Wheaton College.

Does this paper:

1. clearly (re)state the problem to be solved?
2. provide a paragraph which explains how the problem will be approached?
3. state the answer in a few complete sentences which stand on their own?
4. give a precise and well-organized explanation of how the answer was found?
5. clearly label diagrams, tables, graphs, or other visual representations of the math?
6. define all variables, terminology, and notation used?
7. clearly state the assumptions which underlie the formulas and theorems, and explain how each formula or theorem is derived, or where it can be found?
8. give acknowledgment where it is due?
9. use correct spelling, grammar, and punctuation?
10. contain correct mathematics?
11. solve the questions that were originally asked?