Name:

Math 29 – Probability

Practice Second Midterm Exam 1

Instructions:

1. Show all work. You may receive partial credit for partially completed problems.
2. You may use calculators and a one-sided sheet of reference notes. You may not use any other references or any texts.
3. You may not discuss the exam with anyone but me.
4. Suggestion: Read all questions before beginning and complete the ones you know best first. Point values per problem are displayed below if that helps you allocate your time among problems. Note that problem 4 is spread over 2 pages for spacing.
5. You need to demonstrate that you can solve all integrals that do not have a (DO NOT SOLVE) statement. I.E. write out some work showing how you solved the integration, including if necessary integration by parts.
6. Good luck!
7. This exam would either be reduced to 3 questions (likely, dropping question 1), or question 4 would be cut in half.

Problem	1	2	3	4	Total
Points Earned					
Possible Points					50

1. Consider the following Markov chain: We start with 2 urns that, between them, contain four balls. At each step, one of the four balls is randomly selected from one urn and is moved to the other urn. The states will be the number of balls in the first urn. The transition matrix for this Markov chain is given by (you will have to write in the states 0,1,2,3,4 at the edges yourself if you want them:

0	1	0	0	0
.25	0	.75	0	0
0	.5	0	.5	0
0	0	.75	0	.25
0	0	0	1	0

a. Are there any absorbing sets (apart from the entire sample space)? If so, state them.

b. This matrix is (circle one) ***reducible*** ***irreducible*** .

c. If you start with 2 balls in each urn, what is the probability that the first urn has 4 balls in it 2 steps later?

d. Assuming you are equally likely to start in any state, what is the probability you end up with 2 balls in urn 1 in 2 steps?

2. Suppose the diameter at breast height of trees (in inches) of a certain type is normally distributed with mean 8.8 and standard deviation 2.8, based on data in a 1997 article in the Forest Products Journal.

a. What is the probability that the diameter of a randomly selected tree of this type will exceed 10 inches?

b. The 10% of trees of this type with the smallest diameter at breast height have a breast height of ________ or smaller. Fill in the blank using appropriate probability calculations to arrive at a numerical value.

c. Suppose four trees of this type are randomly (independently) selected. What is the probability that at least one has a diameter at breast height exceeding 10 inches?

d. Now suppose 450 trees of this type are randomly (independently) selected. What is the approximate probability that at least 165 of these trees have a diameter at breast height exceeding 10 inches? (Make this approximation the best you can.)

3. Suppose a local bank has opened a new drive-up window. In a single hour of operation, an average of 8 customers uses the window. Suppose you decided to model X, the number of customers using the window in an hour, as a Poisson random variable.

a. How would you model Y, the time between successive arrivals of customers at the drive-up window? (Give distribution and appropriate parameters)

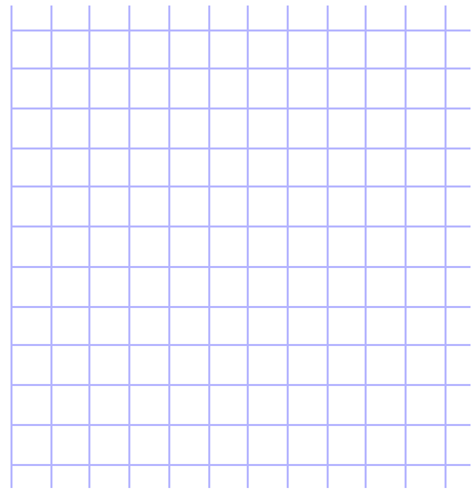
b. How would you model W, the time it takes for 2 customers to arrive at the drive-up window assuming one customer just finished? (Hint: W=Y+Y=2Y.)

c. What is the probability that it takes longer than 15 minutes for 2 customers to arrive at the window assuming one customer just finished?

d. What are the mean and variance for V= sum of 8 times between successive arrivals (i.e. V=8Y)? Explain in one sentence why these make sense given the initial starting distribution was Poisson(8).

4. (Source: Devore/Berk) A nut company sells a deluxe can of mixed nuts that contains almonds, cashews, and peanuts. The net weight of a can is exactly one pound, but the weight contributions of each type of nut can be considered random. Let X= weight of almonds in a selected can, and let Y=the weight of cashews in the selected can (note that if Z = weight of peanuts in the selected can, then X+Y+Z=1). Suppose the joint pdf of X and Y is given by $f(x,y) = 24xy, 0 \le x, y \le 1, 0 \le x + y \le 1$, and 0, otherwise.

a. Sketch and shade the region where the pdf takes positive probability density.



b. What is the probability that cashews and almonds make up at most 50% of the weight of the selected can? (Note that in a "deluxe" mix, you'd expect the fraction of peanuts (Z) to be low).

c. Set up an integral or integrals (do NOT solve) to determine the probability that the weight of cashews is at most twice the weight of almonds in the selected can.

4 (continued)

d. What is the marginal distribution of the weight of cashews?

e. What is the conditional distribution of the weight of almonds given the weight of cashews is .3?

f. Set up an integral (DO NOT SOLVE) to evaluate the probability that the weight of almonds is more than .5 given that the weight of cashews is .3?