Lose the Scratch Ticket, Keep the Gamble: The Appeal of Prize-Linked Savings Under Different Theories of Gambling Behavior

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Submitted to the Department of Economics of Amherst College
In partial fulfillment of the requirements for the degree of
Bachelor of Arts with Honors

Faculty Advisor: Professor Ishii

May 3rd, 2012

Acknowledgements

I would first like to thank my advisor, Professor Ishii, for his guidance throughout the entire thesis process. Without his consistent advice, criticism, and availablility, this thesis would never have been completed.

I also wish to extend my gratitude to Andy Anderson who spent countless hours working with me on my modeling in Mathematica, and Professor Nicholson for his feedback as I was first getting started.

Finally, I want to thank my friends, family, and fellow economics thesis writers for their support and encouragement.

Abstract

Policy makers and economists suggest that low-income households in the U.S. do not save enough. Many of the low-income individuals who tend to save too little also engage in gambling activities more than the general population. As a result, Prize-Linked Savings (PLS) Programs, which combine the prospect of saving and gambling, may help encourage greater savings among low-income individuals. This thesis explores how two leading theories of gambling behavior, subjective probability and indivisible goods, inform the prize structure that maximizes the appeal of PLS programs to the targeted low-income individuals.

I find that the design of PLS programs that maximizes their attractiveness depends largely upon the assumed theory of gambling behavior. Under subjective probability, a "winner take all" prize structure is favored, while under indivisible goods, a prize structure with more modest sizes that still allow indivisible goods consumption is favored. I also consider a model where one portion of the population is assumed to follow subjective probability while the other portion indivisible goods. The trade-off between the two theories suggests that when one accounts for 60% of gambling behavior preferences, this theory dominates the prize structure. My results underscore the need for more empirical research to determine the gambling behavior preferences of the population given that the effectiveness of PLS programs depends mainly on the assumed theory underlying gambling behavior.

Keywords: Quantitative Policy Modeling; Behavioral Economics; Gambling

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1 Introduction

According to many policy makers, low-income households in the U.S. do not save enough. The Corporation for Enterprise Development reports that 43.1% of American households—some 127.5 million people—are "liquid asset poor". Liquid asset poor is defined as lacking the sufficient amount of liquid assets to subsist at the poverty level for three months in the absence of income.[1] This figure suggests that nearly half of all Americans are one emergency away from being able to finance basic expenditures such as food in the near future. Furthermore, a number of economists assert that the lack of savings behavior is especially pronounced in the low-income demographic. A study using the Consumer Expenditure Survey found that the savings rate is roughly -23% among the lowest income quintile in comparison to 46% among the highest income quintile (Dynan et al., 2000, p. 21). As a result of this savings behavior, Duflo et al. (2006) suggest that a significant portion of Americans at this income bracket do not set aside enough money for retirement. These authors cite that households with annual incomes below $40,000 are unlikely to allocate money toward retirement accounts (IRAs), rarely have employer-provided pensions, and in 2001 had a median net financial wealth of $2,200 outside of retirement accounts (Duflo, 2007, p. 647).

In response, policy makers have attempted to increase the savings rate among low-income individuals through various incentives. Bronchetti et al. (2011) examine the impact of an opt-out savings program where low-income tax filers by default receive a portion of their tax refund in the form of U.S. Savings Bonds. Despite these efforts, the

[1] Source: http://scorecard.assetsandopportunity.org/2012/measure/liquid-asset-poverty-rate

savings rate among the low-income demographic continues to be low, especially relative to higher income individuals.

In addition to the low rate of savings among the low-income demographic, and perhaps even contributing to the issue, is the popularity of gambling among individuals in this income bracket. Data show that while low-income individuals tend to save a lower proportion of their income than the population as a whole, they also tend to gamble a higher proportion of their income.[2] Given that traditional savings instruments and pro-savings government policies have not incentivized less wealthy individuals to save enough, but that gambling seems so appealing, one potential solution is Prize-Linked Savings programs. My thesis focuses on how to design Prize-Linked Savings programs to maximize the expected utility from participation and, thus, maximize their attractiveness. I define this design as the "optimal design".

Prize-Linked Savings (PLS) programs are a hybrid between a savings account and a lottery that are targeted toward the low-income gambling demographic who usually do not save. In a typical program, the participants first deposit money in an account just as they would with a standard savings account. They then forgo the interest they would have received in exchange for a stake in a lottery. All of the foregone interest among participants is compiled, and paid out in larger sums as part of a prize structure. The prize structure, or design of the program, can vary in the number of prizes, the probability of prizes, and the size of prizes. Intuitively, these programs expand the utility of low-income individuals because the gambling component makes saving more attractive, and offers a higher utility than a regular savings instrument. The exact extent of this utility

[2] Source: http://lottoreport.com/demostudy2005.htm

gain, however, depends upon the design of the program. This paper studies the optimal design of a PLS program that maximizes the program's attractiveness to low-income individuals under differing theories explaining the gambling behavior of such individuals.

PLS programs have been in existence since 1694 (Kearney et al., 2010, p. 7), and despite being currently outlawed in the U.S., have a large international appeal. Examples of successful programs range from countries such as Mexico and Columbia to Pakistan and Japan. In the U.K., government-sponsored Premium Bonds have been a popular PLS program since the 1950's. This program, with an estimated 40% participation rate among British households, now draws investments of over £40 billion and offers annual jackpots of over £1 million (Kearney et al., 2000, p. 10). In South Africa, the First National Bank introduced the Million a Month Account (MaMA) in 2005. This example, a privately-owned PLS program as opposed to Premium Bonds, offered monthly drawings of 1 million rand (Kearney et al., 2000, p. 12). Despite only running for 3 years, and facing government opposition, MaMA managed to open accounts for over 1% of the unbanked in South Africa (Kearney et al., 2000, p. 13). This is but one example of a PLS program that found high demand from customers who had not saved previously.

Part of the reason that PLS programs have been so successful, especially among low-income households, is the demand for gambling products. Traditional expected utility theory would predict that low-income individuals are less likely to gamble than high-income individuals. Due to the generally accepted principles of diminishing marginal utility of wealth and decreasing relative risk aversion, a given gamble would be more appealing to a wealthier individual than a poorer individual. Nevertheless, empirical lottery data from around the country suggest that the opposite is true. For example, in a study conducted in Texas in 2005, it was found that those earning under $20,000 spend .380% of their income on the lottery while those making between $76,000-$100,000 spend .034%.[3] In practice, then, it appears that the poor display less risk aversion than the wealthy when it comes to lottery consumption. Kearney (2004) demonstrates that with the introduction of a state lottery with instant games ("scratch tickets"), low-income households within that state reduce expenditures on necessary non-gambling products by 3.1% (Kearney, 2005, p. 2271). These products include food and clothing as well as home mortgage, rent, and other bills. This tendency to shift consumption away from necessary goods towards gambling is especially pronounced given that the average scratch ticket is not an actuarially fair gamble and pays out only $0.52 on every dollar (Kearney et al., 2000, p. 4).

In order to understand the appeal of PLS, which lies in the gambling component of the programs, it is necessary to examine various theories explaining gambling behavior among low-income individuals. Two of the leading theories in gambling behavior are subjective probability weighting and indivisible goods. Under the assumption that gambling behavior is explained by these theories, I ask: how do subjective probability weighting and indivisible goods inform the optimal design of PLS programs? Previous studies have only examined these questions under one gambling behavior theory. This paper expands upon the literature by considering the combination of competing gambling behavior theories to simulate heterogeneous gambling behavior preferences in the general population.

[3] Source: http://www.txlottery.org/export/sites/lottery/Documents/demographicreport2004.pdf

I find the optimal design of a PLS program under subjective probability favors a low-probability "winner take all" jackpot while allocating a negligible amount of the prize pool to the other prizes. The optimal design under indivisible goods, however, involves several prizes of more modest sizes still large enough to purchase different indivisible goods. When I combine both theories, the trade-off between the two is such that if I assume that one theory accounts for 60% of the gambling behavior preferences in the population, this theory dominates the prize structure. Given the large effect of the assumed gambling behavior theory on the optimal PLS prize structure, this thesis underscores the need for empirical research to determine the theory underlying gambling behavior in the general population.

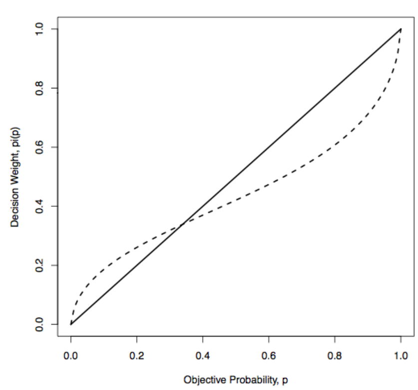
2 Theories on Gambling Behavior

Previous studies on the optimal design of PLS programs focus only around prospect theory. Given that there are many more models of gambling behavior, and consumer preferences likely do not follow one theory alone, this thesis expands upon the existing literature by combining two leading theories of gambling behavior. Then, this thesis examines how the conclusions about the optimal design of PLS programs differ from previous models.

The leading theories of gambling behavior can be separated into two classes: the psychological and behavioral theories, and the alternative utility function theories. Several authors have attempted to explain gambling behavior through the psychological motivations underlying lottery purchase behavior. Thiel (1991) proposed that a lottery ticket is comprised of two bundled goods where the first good is the ticket itself and the second good, considered to be fantasy or hope, entices people to participate in what would normally not be a rational gamble. While this field of study provides a good start in explaining low-income gambling, it lacks a strong empirical foundation. Given that the psychological theories are difficult to quantify, they are also difficult to incorporate into empirically verifiable models.

Tversky and Kahneman (1979) provide an alternative behavioral explanation for low-income gambling with their work on prospect theory and subjective probability. By offering a series of wagers to participants, these authors find that people's revealed preferences seem to contradict expected utility theory. Using the data they collected, Tversky and Kahneman rationalize this behavior using the notion of subjective probability weighting. Subjective probability weighting supposes that there is a nonlinear function, $\pi(p)$, which translates objective probabilities, p, into subjective decision weights. This function is found to be S-shaped, and can be seen below:

Figure 2.1

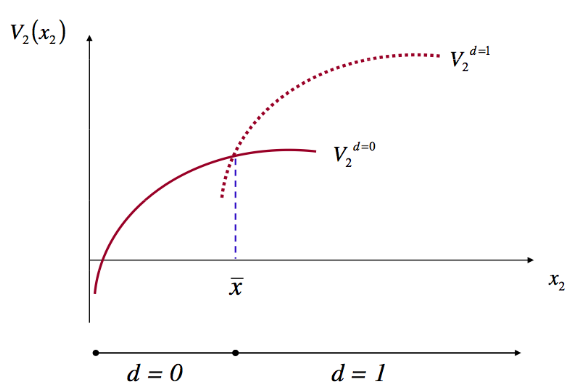


Source: Tverskey and Kahneman (1979)

Under subjective probability weighting, low probabilities are weighted upward while high probabilities are weighted downward. With respect to lotteries, the relevant part of the decision weighting function is the leftmost portion. When individuals overestimate their odds of winning low-probability lotteries, they also overestimate the expected return of each gamble they make. This leads to more gambling than what is objectively rational. Tversky and Kahneman, then, are able to provide an explanation for why many individuals exhibit high levels of lottery consumption.

A second class of explanations rationalizing gambling behavior is based upon kinks in the utility curve. While Friedman and Savage (1948) were the first to introduce an alternative utility function to the traditional model, Yew-Kwang Ng (1965) expanded upon this idea by introducing borrowing constraints and indivisible goods. Indivisible goods are those that may only be purchased discretely, such as a car or house. Ng explores a model where one good, college education, is indivisible while all other goods are perfectly divisible. Due to imperfect capital markets, low-income individuals cannot take out the loans necessary to purchase these indivisible goods. As a result, although the marginal utility of consumption is diminishing, there is a kink in the utility function at the point where the indivisible good is consumed:

Figure 2.2



Source: Crossley et al. (2011)

In Figure 2.2, x_2 is disposable income, $V_2(x_2)$ is the utility function, d = {0,1} is the purchase of the indivisible good, and $\bar{x}$ is the level of income at which this purchase can be made. Due to the convex kink in the utility function at $\bar{x}$, individuals gamble for the opportunity to purchase the indivisible good and move from the first to the second portion of the utility curve.

In this paper, I focus on subjective probability and indivisible goods theories. I do not assert that subjective probability and indivisible goods explain why all gambling occurs, but rather that they provide a foundation for understanding this behavior. My work uses both of these theories to inform the optimal design of PLS programs. Given that existing literature has focused on incorporating one theory into this design, this paper expands upon previous studies by considering both subjective probability and indivisible goods, and determining how the subsequent optimal PLS prize structure differs when balancing the two theories.

3 The Model

I present a model that examines how subjective probability and indivisible goods theories of gambling behavior inform the optimal design of PLS programs. The design depends on three factors: the size of the prizes, the probability distribution of the prizes, and the number of prizes. The model focuses solely upon the size of the prizes, rather than the other two factors, as it is computationally taxing to solve for more than one factor at a time.[4,5] The design problem in this paper focuses on choosing the size of the prizes for a given 5-prize lottery with exogenously assigned probabilities. The assumed probabilities are derived from scratch card data (see appendix), and can be seen below:

[4] Mathematica had difficulty solving for the size of the prizes and the prize probabilities simultaneously.
[5] Section 5 relaxes some of these assumptions to test whether they are robust.

Table 3.1

Prize	Probability
q_1	$p_1 = 1/600,000.00$
q_2	$p_2 = 1/40,000.00$
q_3	$p_3 = 1/1,481.40$
q_4	$p_4 = 1/30.84$
q_5	$p_5 = 1/5.20$
0 (no prize)	$p_6 = 1/1.30$

Section 3.1 examines the optimal design under the assumption that consumer preferences follow subjective probability. Section 3.2 examines the design under the assumption that preferences follow indivisible goods. Finally, Section 3.3 combines the two models.

3.1 Subjective Probability

I follow Pfiffelmann (2007) in my model of the optimal design of a 5-prize PLS program for consumers whose gambling behavior is explained by subjective probability. These consumers map objective probabilities into subjective probabilities, and for this reason, have preferences for prizes with certain probabilities. Given that small probabilities are overweighted, consumers who follow prospect theory favor an allocation of the majority of the prize pool to the lowest-probability prize. While the model also incorporates some aspects of prospect theory in order to remain consistent with the comprehensive theory presented by Tversky and Kahneman, what is relevant in rationalizing gambling theory is subjective probability.

Given a particular lottery with a set of payoffs, an individual perceives this joint set of probabilities as a prospect, X. Prospect X can be defined as:

$$X = ((x_i,p_i)i = -m,...n) \tag{1}$$

with $x_{-m} < x_{-m+1} < ... < x_0 = 0 < x_1 < x_2 < ... < x_n$

In equation 1, x_i is equal to the payout of each prize, and p_i is equal to the probability of that payout occurring. Each pair (x_i,p_i) is one outcome of the prospect. Within this prospect, people perceive two different types of outcomes. Certain payouts are considered gains, $x_n > 0$, while others are considered losses, $x_{-m} < 0$, relative to a reference point, r_f. This reference point corresponds to the return that one expects to receive, such as the return on a standard savings account or low-risk investment. Due to risk aversion, the valuation of gains and losses is asymmetric, with separate value functions for each outcome. For example, the positive value derived from a gain of $100 would be smaller in magnitude than the negative value derived from a loss of $100. The subjective probability weighting, as well, depends on whether the prize is a gain or a loss.

The value of a prospect is comprised of two components due to the asymmetric valuation of gains and losses. I define $U_{sp}(X^+)$ as the sum of the value from all gains, $U_{sp}(X^-)$ as the sum of the value of all losses, and $U_{sp}(X)$ as the value of the prospect:

$$U_{sp}(X) = U_{sp}(X^+) + U_{sp}(X^-) \tag{2}$$

In this equation, X^+ represents all $x_n > 0$ and X^- represents all $x_{-m} < 0$. Both $U_{sp}(X^+)$ and $U_{sp}(X^-)$ can then be expanded as a subjective expected value of the event occurring, π_i, multiplied by the value of the prize measured by the value function, $v(x_i)$. π_i maps objective probabilities to their subjective values, and $v(x_i)$ accounts for diminishing marginal returns and loss aversion. As both of these functions differ by the type of outcome, a positive superscript indicates that the function is used to evaluate gains, while a negative superscript is used for losses. The expected value of $U_{sp}(X^+)$ and $U_{sp}(X^-)$, respectively, becomes:

$$E(U_{sp}(X^+)) = \sum_{x=0}^{n} (\pi_n{}^+)v^+(x_n) \quad (3)$$

$$E(U_{sp}(X^-)) = \sum_{x=-m}^{0} (\pi_{-m}{}^-)v^-(x_{-m}) \quad (4)$$

$v(x_i)$ is a strictly increasing function that is defined relative to the point $v(0) = 0$.

For both value functions, $v^+(x_n)$ and $v^-(x_{-m})$, Tversky and Kahneman suggest the following functional form:

$$v^+(x_n) = x_n{}^\alpha \quad (5)$$

$$v^-(x_{-m}) = -\lambda(x_{-m})^\beta \quad (6)$$

For $0 < \alpha < 1$ and $0 < \beta < 1$, the value function is convex over gains, and convex over losses. The level of loss aversion in $v^-(x_{-m})$, creating the different weighting of gains and losses, is measured by the parameter λ. Based off experimentation, the authors estimate the parameter values to be $\alpha = \beta = 0.88$ and $\lambda = 2.25$.

The weighting functions, $\pi_n{}^+$ and $\pi_{-m}{}^-$, are defined by Tversky and Kahneman to cumulatively weigh all of the probabilities of a given prospect. As such, the functions are defined as follows:

$$\pi_n{}^+ = w^+(p_n) = w^+[p_i + \ldots + p_n] - w^+[p_{i+1} + \ldots + p_n] \text{ with } 0 \leq i \leq n-1 \quad (7)$$

$$\pi_{-m}{}^- = w^-(p_{-m}) = w^-[p_{-m} + \ldots + p_i] - w^-[p_{-m} + \ldots + p_{i-1}] \text{ with } -m \leq i \leq 0 \quad (8)$$

Through experimentation, Tversky and Kahneman propose the functional form below for the functions $w^+(p_n)$ and $w^-(p_{-m})$:

$$w^+[p_n] = \frac{p^{\gamma+}}{(p^{\gamma+} + (1-p)^{\gamma+})^{1/\gamma+}} \quad (9)$$

$$w^-[p_{-m}] = \frac{p^{\gamma-}}{(p^{\gamma-} + (1-p)^{\gamma-})^{1/\gamma-}} \quad (10)$$

For $\gamma < 1$, this functional form leads to an s-shaped weighing of objective probabilities where low-probability events are overweighted and high-probability events are underweighted. This indicates that the marginal changes in subjective probability weighting are not constant and that more weighting occurs at high and low objective probabilities. The values of γ^+ and γ^- account for different levels of subjective probability weighting for gains and losses. Tversky and Kahneman estimate $\gamma^+ = .61$ and $\gamma^- = .69$.

Given the weighting functions and the value functions, I now present the lottery and maximization problem underlying design. In this model, the agent is first faced with the decision of whether or not to participate in the PLS program, L = {0,1}, given the 5-prize prize structure above. This decision will depend upon the attractiveness of the prize structure and the expected utility from participation. The size of the deposit necessary to participate in the PLS program is equal to g, while the reference point of return is equal to r_f. The expected payout of the prize structure then is r_f*g, assuming the payout is actuarially fair, and prizes are perceived as gains or losses relative to this value. Therefore, all payouts greater than r_f*g will be measured by the value function $v^+(x_n)$, while those less than r_f*g will be measured by the value function $v^-(x_{-m})$.

To design the program, it is necessary to define which prizes are gains and which are losses. Ideally, the model itself would solve this problem such that the optimal value of each prize, q, would determine whether it is a gain or loss. Due to the fact that this creates a non-smooth objective function, however, the maximization becomes computationally taxing if gains and losses are not determined exogenously. Therefore, based upon scratch ticket data, I have chosen the first four prizes, q_1, q_2, q_3, and q_4, to be gains, and the last, q_5, a loss (see appendix). Optimal PLS structure is the one that maximizes the expected utility from participation using the subjective probability weighting and value functions. The maximization problem becomes:

$$\text{Maximize}_{q1,q2,q3,q4,q5}\ E(U_{sp}) = \sum_{n=1}^{4}(\pi^{+}{}_{n})v^{+}(x_n) + \sum_{n=5}^{6}(\pi^{-}{}_{n})v^{-}(x_n)$$

$$\begin{aligned}
= \ & (w^{-}[p_6]) * v^{-}[- r_f * g] + \\
& (w^{-}[p_6 + p_5] - w^{-}[p_6]) * v^{-}[q_5 - r_f * g] + \\
& (w^{+}[p_1 + p_2 + p_3 + p_4] - w^{+}[p_1 + p_2 + p_3]) * v^{+}[q_4 - r_f * g] + \\
& (w^{+}[p_1 + p_2 + p_3] - w^{+}[p_1 + p_2]) * v^{+}[q_3 - r_f * g] + \\
& (w^{+}[p_1 + p_2] - w^{+}[p_1]) * v^{+}[q_2 - r_f * g] + \\
& (w^{+}[p_1]) * v^{+}[q_1 - r_f * g]
\end{aligned}$$

subject to:

$$q_1 * p_1 + q_2 * p_2 + q_3 * p_3 + q_4 * p_4 + q_5 * p_5 = r_f * g$$

$$q_1 - r_f * g \geq 0,\ q_2 - r_f * g \geq 0,\ q_3 - r_f * g \geq 0,\ q_4 - r_f * g \geq 0,\ q_5 \geq 0$$

The first constraint specifies the expected payout of the program, while the second constraint guarantees that all of the winning prizes offer a return greater than the reference point, r_f. The values for $r_f = 0.31$ and $g = 160$ were calibrated using current market data and existing PLS program data (see appendix). In addition, it is assumed that there are $n = 600{,}000$ participants so that q_1 is paid out once (see appendix). The resulting values for q_1, q_2, q_3, q_4, and q_5 determine the optimal PLS prize structure that maximizes the expected value from participation under subjective probability.

3.2 Indivisible Goods

I adopt Crossley et al. (2011) in my model of the optimal design of a 5-prize PLS program under the assumption that consumers follow indivisible goods theory. Given that Crossley et al. focus solely upon the size of the income effects for low-income individuals who play the lottery, my model extends the theory to a utility maximization problem to design PLS programs. This model incorporates the idea that consumer preferences are represented by a utility function with kinks at each point where an indivisible good can be purchased. These consumers have a preference for prizes that are large enough to allow them to purchase additional indivisible goods. As a result, consumers who follow indivisible goods theory are more interested in multiple modest prizes that allow indivisible goods consumption rather than one large jackpot.

Within this model, there are two types of goods: divisible goods, x_1, and indivisible goods, x_2. Divisible goods are defined as food, clothing, and other goods purchased in divisible amounts. Given that these goods are necessary to survival, they are consumed by all individuals. Indivisible goods, alternatively, are goods that can only be consumed discretely. These are more expensive goods that are not available to all individuals. While there is a large array of indivisible goods, the ones included in this model are a car, a home, and a college education such that $x_2 = \{0, 1, 2, 3\}$(see appendix). The price of these goods, $c(x_2) = \{0, c_{1,} c_2, c_3\}$, and the utility from these goods, $d(x_2) = \{0, d_1, d_2, d_3\}$ (see appendix), are represented by the following parameters:

Table 3.2

	Utility Parameter	**Price ($)**
No Indivisible Goods ($x_2 = 0$)	0	0
Car ($x_2 = 1$)	$d_1 = 35$	$c_1 = 13{,}105$
Car + Home ($x_2 = 2$)	$d_2 = 45$	$c_2 = 33{,}518$
Car + Home + College ($x_2 = 3$)	$d_3 = 55$	$c_3 = 48{,}532$

For the purpose of this model, the consumers of the PLS program are modeled as a representative agent. The agent is assumed to be the average lottery consumer with regard to socioeconomic background and indivisible goods owned. For this model, the representative agent has an annual income after taxes, y_0, of \$30,000, does not have access to credit markets, and does not own a car, home, or have a college education (see appendix). In addition, it is assumed that the representative agent sets aside a certain level of income, z_0, toward x_1 to purchase food, clothing, and other items of sustenance. This level of sustenance is utilized for the sake of calculating disposable income, h_0, and is equal to $y_0 - z_0$. For this model, z_0 = \$23,000 such that h_0 = \$7,000 (see appendix).

In the first stage of this two-stage model, the agent is faced with the decision of whether or not to participate in the PLS program, L = {0,1}, given the prize structure. In the second stage, the outcome of the lottery is realized, and the agent decides how to allocate income in the second stage, y_1, between divisible and indivisible goods. The utility of the representative agent, $U_{ig}(x_1,x_2)$, is represented by a Cobb-Douglas utility function:

$$U_{ig}(x_1,x_2) = x_1^{\delta} * (w(x_2))^{\varepsilon} \tag{11}$$

$$\text{where } w(x_2) = \begin{cases} \text{if } x_2 = 1, & 1{,}500 \\ \text{if } x_2 = 2, & 14{,}750 \\ \text{if } x_3 = 3, & 37{,}500 \end{cases}$$

In this equation, x_1 defines the amount of divisible goods consumed, $w(x_2)$ defines the value of each level of indivisible goods where w(0) = 0 (see appendix), and δ and ε are weighting parameters. This utility function incorporates the two-good model where the representative agent allocates money between divisible goods and indivisible goods. While x_1 can be purchased in small quantities at a price P_1, x_2 may only be purchased discretely at price $c(x_2)$. In this model, the values chosen for the weighting parameters, δ and ε, are 0.6 and 0.4, respectively (see appendix).

In order to solve the maximization problem to determine the optimal PLS structure for indivisible goods, this model uses an indirect utility function rather than the direct utility function presented above. Following the work on the two-stage budgeting process (Strotz, 1957; Gorman, 1959), the agent faces an analogous situation in budgeting between x_1 and x_2. It is assumed that the agent first allocates income between these two types of goods. Afterward, the agent allocates within each type of good, or in the case of x_2, which indivisible goods to purchase. For this reason, this model lends itself more naturally to an indirect utility function that can be derived from Equation (11).

Solving the direct utility function for the optimal level of allocation between x_1 and x_2 (see appendix) yields the following indirect utility function:

$$V_{ig}(y_1) = \ln(y_1 - c[h_1]) + a[h_1] \tag{12}$$

where

$$a[h_1] = \begin{cases} \text{if } c_1 < h_1 \le c_2, & b * \left(d_1 + e * {d_1}^2\right) \\ \text{if } c_2 < h_1 \le c_3, & b * \left(d_2 + e * {d_2}^2\right) \\ \text{if } c_3 < h_1, & b * \left(d_3 + e * {d_3}^2\right) \end{cases} \qquad c[h_1] = \begin{cases} \text{if } c_1 < h_1 \le c_2, & c_1 \\ \text{if } c_2 < h_1 \le c_3, & c_2 \\ \text{if } c_3 < h_1, & c_3 \end{cases}$$

y_1 is income and h_1 is disposable after the PLS lottery is realized where $h_1 = y_1 - z_0$. $a[h_1]$ and $c[h_1]$ are the indirect utility and cost functions for indivisible goods, respectively, where $a[0] = c[0] = 0$. $b = 11.9$ and $e = 11.9$ are utility coefficients.

The intuition behind these specifications is that once the disposable income of the agent is high enough, the utility gain from purchasing the indivisible good outweighs the utility gain from only consuming divisible goods. For example, it is desirable to purchase a car when $c_1 < h_1 \le c_2$, but not when $h_1 < c_1$. The agent, then, faces diminishing marginal returns with regard to divisible goods, but has kinks in the indirect utility function at each level of income where an indivisible good is purchased.

The optimal PLS structure under indivisible goods maximizes the attractiveness of the program. This is accomplished by maximizing the expected value of participation:

$$\text{Maximize}_{q1,q2,q3,q4,q5}\ E(V_{ig}) = \sum_{n=1}^{6} p_n(\ln(y_0 + q_n - c[h_0 + q_n]) + a[h_0 + q_n])$$

$$\begin{aligned} = \ & p_1(\ln(y_0 + q_1 - c[h_0 + q_1]) + a[h_0 + q_1]) + \\ & p_2(\ln(y_0 + q_2 - c[h_0 + q_2]) + a[h_0 + q_2]) + \\ & p_3(\ln(y_0 + q_3 - c[h_0 + q_3]) + a[h_0 + q_3]) + \\ & p_4(\ln(y_0 + q_4 - c[h_0 + q_4]) + a[h_0 + q_4]) + \\ & p_5(\ln(y_0 + q_5 - c[h_0 + q_5]) + a[h_0 + q_5]) + \\ & p_6(\ln(y_0 - c[h_0]) + a[h_0]) \end{aligned}$$

subject to:

$$q_1 * p_1 + q_2 * p_2 + q_3 * p_3 + q_4 * p_4 + q_5 * p_5 = r_f * g$$

$$q_1 \geq 0, q_2 \geq 0, q_3 \geq 0, q_4 \geq 0, q_5 \geq 0$$

The constraints specify the expected payout of the program and that no prize may be negative. Each value of y_1 represents the level of income reached depending upon which prize is won. For instance, if q_2 is won with probability p_2, $y_1 = y_0 + q_2$. The same intuition follows for each value of h_1.

3.3 Combined Model

Under the assumption that the optimal design of the 5-prize PLS structure is informed by both subjective probability and indivisible goods, the model incorporates the idea that the agent values how the prize pool is allocated based off the probability as well as the size of the prizes. The preferences of the agent are represented by the parameter $0 < \rho < 1$ where the subjective probability and indivisible goods models are valued with weight ρ and $(1-\rho)$, respectively. This model may be interpreted as representing the gambling behavior preferences of the population as a whole. Changing the value of ρ presents a variety of scenarios where different proportions favor subjective probability or indivisible goods. For this model, the maximization problem becomes:

$$\text{Maximize}_{q1,q2,q3,q4,q5}\ E(U_{cm}) = \rho\, E(U_{sp}) + (1- \rho)\, E(V_{ig})$$

subject to:

$$q_1 * p_1 + q_2 * p_2 + q_3 * p_3 + q_4 * p_4 + q_5 * p_5 = r_f * g$$

$$q_1 - r_f * g \geq 0,\ q_2 - r_f * g \geq 0,\ q_3 - r_f * g \geq 0,\ q_4 - r_f * g \geq 0,\ q_5 \geq 0$$

The second constraint specifies that q_1, q_2, q_3, and q_4 are gains to account for the subjective probability portion of the maximization. It is worth noting that this constraint was not in place with just the indivisible goods model, but it is necessary when the two models are combined. In addition, the parameters b and e from the indivisible goods model were calibrated so that, for each model's respective optimal prize structure, the expected utility from participation is equal. By allowing the base case for each model to provide the same expected utility, this minimizes the extent to which either theory dominates the combined model due to differences in the two utility functions. With this specification, the resulting values of q_1, q_2, q_3, q_4, and q_5 represent the prize structure for the PLS program that maximizes the program's attractiveness where preferences are split between subjective probability and indivisible goods.

4 Results

This thesis presents the optimal design of PLS programs under several gambling behavior theories given the constraints of the payout. The results demonstrate that the assumed gambling behavior theory has a large impact upon the optimal prize structure. Under subjective probability, a “winner take all” prize structure is favored with little allocation toward the other prizes. Under indivisible goods, however, a prize structure with more modest prize sizes is favored to allow additional indivisible goods to be consumed. When the population’s preferences are split between the two theories, the trade-off is such that either subjective probability or indivisible goods dominates the prize structure with a 60% weighting.

4.1 Subjective Probability Results

Under the assumption that gambling behavior is completely informed by subjective probability, the following prize structure presents the optimal PLS design:

Table 4.1

	q_1	q_2	q_3	q_4	q_5
Size ($)	2,875,830	113	5	5	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	96.63%	0.06%	0.07%	3.24%	0%

“Size” is the size of the prize, “Tickets” is the number of winning tickets for the prize, and “% Prize Pool” is the percentage of the prize pool allocated toward this prize. The prize structure above is asymmetric in that almost the entire prize pool is allocated to q_1, 96.63%, in a “winner take all” system, while the remaining prizes receive little allocation. These results are explained by differences between the probabilities in the constraint function and the perceived probabilities in the objective function; even though the primary constraint of the expected payoff of the program (equal to r_f*g) is defined with objective probabilities, the weight that households place upon each outcome of the PLS lottery is based upon subjective probabilities**.** For example, the perceived chance of losing, π_6 = 58.687%, is less than the actual chance of losing, p_6 = 77.457%, while the perceived chance of winning q_2, π_2 = .155%, is greater than the actual chance of winning the q_2, p_2 = .003%. To maximize $E(U_{sp})$ while maintaining an expected payout of r_f, the optimal prize structure exploits disparities between objective and subjective probabilities. Since the agent inaccurately perceives that the lottery offers a higher expected return by shifting resources from higher to lower probability prizes, the majority of the prize pool is placed in q_1 while the remaining prizes offer approximately the minimum return under the constraints. Therefore, under subjective probability theory, the agent prefers a "winner take all" prize structure.

4.2 Indivisible Goods Results

Assuming that gambling behavior is completely informed by indivisible goods, the optimal PLS prize structure becomes:

Table 4.2

	q_1	q_2	q_3	q_4	q_5
Size ($)	41,699	26,669	6,257	0	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	1.40%	13.44%	85.16%	0%	0%

In this scenario, the prize structure is different than what we find under subjective probability theory. Under indivisible goods, the agent prefers more modestly sized prizes that allow additional indivisible goods to be consumed. This preference is split between the level of indivisible goods consumption made possible with the prize (car vs car and home), and the number of winning tickets for the prize allowing some indivisible goods consumption. I define these two preferences "quality and quantity" of indivisible goods consumption. There is a natural tradeoff between quality and quantity because as money is allocated to prizes allowing a higher quality of indivisible goods to be consumed, there will be a lower quantity of tickets allowing some indivisible goods consumption. This is a choice between locking-in a higher probability of reaching a lower kink in the indirect utility function, or gambling for the chance to reach a higher kink. For instance, the prize structure above offers prizes that allow 405 cars, 15 cars and homes, and 1 car, home, and college education to be consumed with q_3, q_2, and q_1, respectively. Instead, the prize structure could offer 16 tickets to consume all three indivisible goods between q_1 and q_2, while 0 tickets to consume just the car or car and home. Balancing quality and quantity of indivisible goods consumption is an empirical question that depends on the probabilities of the prizes, the size of the prize pool, and the marginal utility of each kink.

Under the preference of indivisible goods, instead of allocating a large portion of the prize pool toward q_1, this prize is only slightly more than the minimum amount necessary to purchase the car, the home, and a college education. Given that h_0 = \$7,000, and that all three combined goods cost \$48,532, a \$41,532 prize is necessary to make this purchase feasible. Intuitively it makes sense that this prize is not much larger than \$41,532 because after this kink in the utility function, additional prize money won from this prize is allocated to divisible goods and subject to diminishing marginal returns in utility.

The second prize, q_2 = \$26,669, allows 15 cars and homes to be consumed. By limiting the size of q_2, it is possible for q_3 = \$6,257 to be large enough for 405 cars to be consumed. Each of the top three prizes, then, allows the consumer to be on a different portion of the indirect utility function. Under the current parameters, the preference for the quantity of indivisible goods consumption is greater than the preference for quality.

The remaining prize pool is distributed between q_4, and q_5. The allocation to these prizes is negligible. The majority of the prize pool is divided among q_1, q_2 and q_3. This underscores the idea that under indivisible goods theory, the agent is interested in several prizes that allow indivisible goods to be consumed. Therefore, an agent under indivisible goods has a preference for more modest prizes than the "winner take all" system under subjective probability.

4.3 Combined Model Results

Under the assumption that both subjective probability and indivisible goods factor into gambling behavior, the optimal PLS design is informed by both theories. This model more realistically reflects the real world scenario where some people gamble due to subjective probability, while others due to indivisible goods. A person who prefers subjective probability will derive less utility from the optimal indivisible goods design, while a person who prefers indivisible goods will derive less utility from the optimal subjective probability design. This idea is presented in the table below:

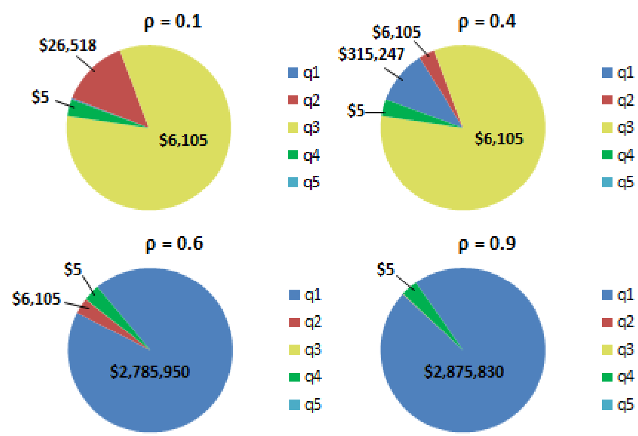
Table 4.3

	$E(U_{sp})$	$E(V_{ig})$
Optimal SP	135.1	11.0
Optimal IG	26.6	135.1

"Optimal SP" indicates that the prize structure in Table 4.1 was plugged into each utility function, while "Optimal IG" indicates the prize structure in Table 4.2 was used. While both the indivisible goods agent and the subjective probability agent have the same expected utility for their corresponding optimal prize structures, $E(U_{sp}) = E(V_{ig}) = 135.1$, both agents have much lower expected utility for the opposite optimal prize structure. As a result, to design a program that considers both theories, the prize structure must find a compromise.

The resulting prize structure ultimately depends upon ρ, the weighting of the two theories. While it is difficult to estimate this parameter precisely, it is possible to contrast how the optimal PLS structure varies depending on different assumptions about the population's preferences for gambling behavior. Presented below in Figure 4.1 are scenarios ranging from a heavy weighting of subjective probability to a heavy weighting of indivisible goods:

Figure 4.1



Depending on the value of ρ, the prize structure more closely resembles that of subjective probability or indivisible goods. The area of each slice represents the percentage of the prize pool that is allocated toward each prize, while the label denotes the size of the prize. For example, for $\rho = 0.40$, q_3 =$6,105 makes up approximately 83% of the prize pool with the 405 tickets paid out for this prize. This figure shows the trade-off between preferences for one large prize in comparison several prizes that allow the consumption of indivisible goods.

When $\rho = 0.1$, the prize structure is qualitatively similar to the indivisible goods prize structure. The majority of the prize pool is allocated to q_2 = \$26,518 and q_3 = \$6,105 to allow the consumption of 15 cars and homes and 405 cars, respectively. The main difference between the two scenarios is that in the combined model, the constraint on q_4 as a gain shifts allocation away from q_1 to q_4. As a result, q_1 = \$9,052 only allows the car to be consumed. Due to the fact that the payout to this prize is now so small, it does not show up in Figure 4.1. The smaller allocation to q_1, rather than q_2 or q_3, demonstrates the preference for quantity of indivisible goods over quality. In the $\rho = 0.4$ scenario, the stronger preference for subjective probability leads to a larger allocation to q_1 = \$315,247 away from q_2. As a result, q_2 is only large enough to allow the car to be consumed. Nevertheless, q_2 and q_3 still account for the majority of the prize pool, indicating that the effect of indivisible goods dominates.

In the $\rho = 0.6$ scenario, the prize structure more closely resembles the $\rho = 1$ case. With a larger preference for subjective probability, q_1 = \$2,785,950 accounts for almost 94% of the prize pool. This follows from the preference under subjective probability for a "winner take all" prize structure. Due to the 40% preference for indivisible goods, the allocation to q_1 is limited such that q_2 = \$6,105 is large enough to purchase the car. When the weighting for subjective probability increases to $\rho = 0.9$, however, no indivisible goods may be purchased with q_2 and q_3. The resulting prize structure is identical to when $\rho = 1$.

To demonstrate the trade-off between the two theories, the expected utility of each agent from participation is presented for the four scenarios:

Table 4.4

	$E(U_{sp})$	**$E(V_{ig})$**
$\rho = 0.1$	23.7	134.7
$\rho = 0.4$	35.9	132.3
$\rho = 0.6$	133.9	15.4
$\rho = 0.9$	135.1	11.0

The trade-off between a prize structure dominated by subjective probability or indivisible goods has a large impact on the expected utility of each type of agent from participation. For both agents, a prize structure dominated by the opposite theory creates a discrete fall in expected utility. Interestingly, if $\rho = 0.5$ and equal shares of the population have preferences for subjective probability and indivisible goods, the optimal prize structure is identical to the $\rho = 0.4$ scenario. This suggests that, in deviating from a prize structure dominated by each respective theory, the fall in marginal utility for indivisible goods is greater than the fall in marginal utility for subjective probability. This is consistent with the fact that $E(V_{ig}) = 15.4$ for $\rho = 0.4$ is less than $E(U_{sp}) = 35.9$ for $\rho = 0.6$.

The assumptions concerning the influence of subjective probability and indivisible goods theories in gambling behavior have strong implications on the optimal PLS design. When there is a larger weighting for subjective probability, the prize pool resembles a "winner take all" system. When there is a larger weighting for indivisible goods, however, there an emphasis on allocation away from the lowest-probability prize so that these goods may be consumed. Thus, by allowing the population to have different preferences for the two theories, we see that the resulting prize structure is a compromise, but that a 60% preference for either theory dominates the prize structure.

5 Robustness

The quantitative results of Section 4 depend on the calibrated parameters for each of the models. But the main purpose of this thesis is not to give specific predictions, but rather to provide a qualitative result of how subjective probability and indivisible goods theories inform the optimal design of PLS programs. In this section, I relax some of the assumptions made about the parameters, and assess to see the extent to which my results are robust to alternative calibrations.

5.1 Subjective Probability Robustness

In the subjective probability model, several assumptions are made about the prizes as gains and losses and the parameters of the weighting functions that have different effects on the results. Since the choice of prizes as gains and losses determines the constraints of the maximization problem, it also has a direct impact on the prize pool distribution. The weighting parameters, alternatively, determine the extent to which objective probabilities are mapped subjectively. Given that these parameters are important factors in the results of the model, I test the robustness of the original results by allowing these parameters to change and re-solving for the optimal prize structure.

5.1.1 Number of Gains

I now allow the number of prizes that are gains, with r_f*g as a reference point, to increase from 4 to 5 prizes and decrease from 4 to 3 prizes. This considers the possibility of a lottery that pays out a different amount of prizes with a higher return than a standard savings account or low-risk investment.

Re-solving for the 5 gains case, the optimal PLS structure becomes:

Table 5.1

5 Gains	q_1	q_2	q_3	q_4	q_5
Size ($)	2,303,850	91	5	5	5
Tickets	1	15	405	19,449	115,385
% Prize Pool	77.41%	0.05%	0.07%	3.24%	19.23%

The additional prize, q_5, receives the minimum allocation possible for it to be a gain relative to r_f*g. The portion of the prize pool allocated to q_5 is taken away from q_1, while the other prizes remain approximately the same. Under subjective probability, the representative agent still prefers a "winner take all" structure, which explains why q_5 is as small as possible. Re-solving instead for the 3 gains scenario, the optimal prize structure follows the same intuition (see appendix). As a result, we can conclude that the subjective probability qualitative result does not vary by changing the number of gains and losses.

5.1.2 Weighting Parameters

I now allow $\gamma^+ = 0.61$, the parameter used to measure the weighting of probabilities of gains, $w^+(p_n)$, to vary by 0.05. Given that Tversky and Kahneman calculated this parameter, adjusting it accounts for sampling error in their estimates.

By increasing γ^+ to $\gamma^+ = 0.66$, the following prize structure results:

Table 5.2

γ^+ = .66	q_1	q_2	q_3	q_4	q_5
Size ($)	2,870,850	453	5	5	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	96.46%	0.23%	0.07%	3.24%	0%

Due to the functional form of $w^+(p_n)$, there is an inverse relationship between γ^+ and the subjective weighting of probabilities; when γ^+ increases, the overweighting of small probabilities is lower. As a result, there is a smaller disparity between subjective and objective probabilities, and it is less attractive to allocate resources to q_1. Nevertheless, the prize structure for $\gamma^+ = .66$ provides the same "winner take all" result. Alternatively, if γ^+ is decreased by 0.05 to $\gamma^+ = 0.56$, the level of subjective probability weighting increases such that more of the prize pool is allocated toward q_1 (see appendix).

While the robustness checks for γ^- are not shown, the same intuition holds for this parameter. Increasing γ^- leads to a smaller underweighting of high probability losses, making it more attractive to allocate the prize pool toward these prizes. The empirical differences when adjusting this parameter are small, however, just as with γ^+. We conclude, then, that the model is robust to modest changes in these weighting parameters.

5.2 Indivisible Goods Robustness

Key assumptions in the indivisible goods model that affect the optimal PLS structure include the number of indivisible goods, the disposable income of the agent, and the parameters of the indirect utility function. The number of indivisible goods impacts the tiers of prize sizes the agent desires, disposable income determines the prize sizes necessary consume indivisible goods, and the indirect utility function parameters determine the preference for the goods. Several of these assumptions are relaxed and the PLS structure is re-solved as a robustness test of the indivisible goods results.

5.2.1 Number of Kinks

I now allow for the number of kinks in the indirect utility function, $V_{ig}(y_1)$, to increase to 4 and decrease to 2. Allowing the number of kinks to change accounts for the possibility that the agent prefers an additional indivisible good ($x_2 = \{0, 1, 2, 3, 4\}$), or the agent only wants the car and home without the college education ($x_2 = \{0, 1, 2\}$).

Assuming $c_4 = \$70,000$ and $d_4 = 65$, the 4-kink optimal prize structure is:

Table 5.3

4 Kinks	q_1	q_2	q_3	q_4	q_5
Size ($)	63,101	26,619	6,206	0	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	2.12%	13.42%	84.46%	0%	0%

Due to the presence of the fourth indivisible good, q_1 increases approximately the minimum amount necessary to allow this good to be purchased. While the shift toward q_1 leads to a small decrease in the allocation toward q_2 and q_3, these prizes are still large enough to consume the car and car and home, respectively.

When the number of kinks is instead decreased from 3 to 2, q_1 is now only large enough to consume the car and home, the most expensive indivisible good (see appendix). As a result, the allocation toward q_2 and q_3 is greater, but the prize structure is qualitatively the same. Therefore, the results found for the indivisible goods model in Section 4 are robust to changing the number of kinks in the indirect utility function.

5.2.2 Disposable Income

While holding the number of kinks constant, I now allow for the level of disposable income to vary by $5,000. The results that follow for h_0 = $12,000 (+$5,000) and h_0 = $2,000 (-$5,000) are intuitive. In increasing h_0 by $5,000, the size of the prizes necessary to purchase each level of indivisible goods are $5,000 smaller, allowing for more winning tickets (see appendix). Given that the extra prize pool is still not large enough for q_2 or q_3 to allow a higher quality indivisible good to be purchased, this money is allocated instead toward q_4 and q_5 instead. If I allow disposable income to decrease by $5,000 to h_0 = $2,000, the prize pool is no longer large enough to allocate to q_3 the amount necessary to purchase a car (see appendix). Given that there are fewer tickets that allow indivisible goods to be purchased, the agent prefers the opportunity to consume higher quality indivisible goods from q_1 and q_2. For this reason, both prizes are just large enough to consume all three indivisible goods, while the remaining prize pool is allocated among q_3, q_4, and q_5.

5.2.3 Indirect Utility Function Parameters

I now allow the parameters in the indirect utility function, $d(x_2)$, to change in order to vary the levels of marginal utility the representative agent derives from each indivisible good. We might expect that an alternative specification would affect the trade-off between quality and quantity. While the parameters in Section 4 were defined such that the car provided the largest kink in the indirect utility function, I now solve for the case where the marginal increase from all three indivisible goods is the largest with $d_1 = 15$, $d_2 = 35$, and $d_3 = 55$. This implies a marginal increase in utility approximately eight times larger for the car, home, and college education than the car alone, and approximately one and a half times larger than the car and home. In this scenario, the prize structure becomes:

Table 5.4

Parameters	q_1	q_2	q_3	q_4	q_5
Size ($)	41,686	26,672	6,256	0	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	1.40%	13.44%	85.16%	0%	0%

Despite the alternative specification, the prize structure is still nearly identical to the original result. Instead of allocating more toward q_2 to give the maximum opportunity to consume all three indivisible goods and gambling on a higher quality of indivisible goods consumption, the consumer settles for a higher quantity of indivisible goods consumption. Consequently, the results found in Section 4 are robust to alternative specifications of the utility parameters.

5.3 Combined Model Robustness

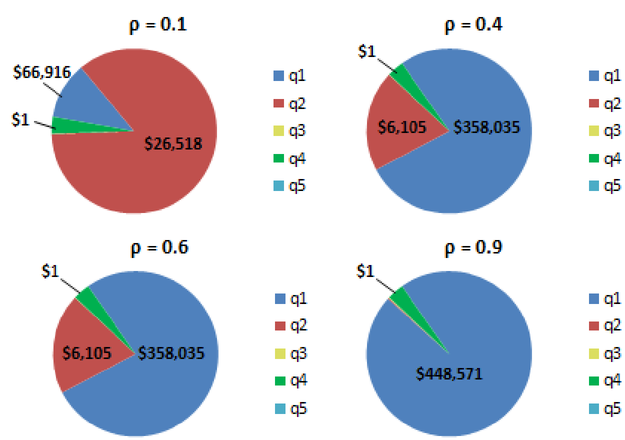
In subjective probability, indivisible goods, and combined models, I made several assumptions about the scale of the PLS program, the number of prizes, and the prize probabilities. Given that the scale of the program directly affects the prize pool, and that the number of prizes and probabilities are two of the three factors in an optimal design, it is important to test the robustness of the results to changes in these parameters.

5.3.1 Prize Pool Size

I now allow for the cost of participation in the PLS program, g, to vary. Given that the model was based off of a program similar in scale to the Premium Bond program in the U.K., this scenario more accurately reflects the current situation in the U.S. where PLS programs are limited in scale. Based off of the Save to Win Program in Michigan, I now allow g = $25, the minimum deposit, decreasing the prize pool by more than 5 fold.[6]

With this new specification, the following prize structures result for the same world scenarios from Section 4:

Figure 5.1



[6] Source: http://www.savetowin.org/

With a smaller prize pool, q_3 cannot pay out enough to consume an indivisible good. The trade-off between the quality and quantity of indivisible goods consumption is affected since a higher quantity of goods is no longer available. Instead, the prize structures presented for different values of ρ vary in the allocation between q_1 and q_2. Otherwise, however, the results are qualitatively the same.

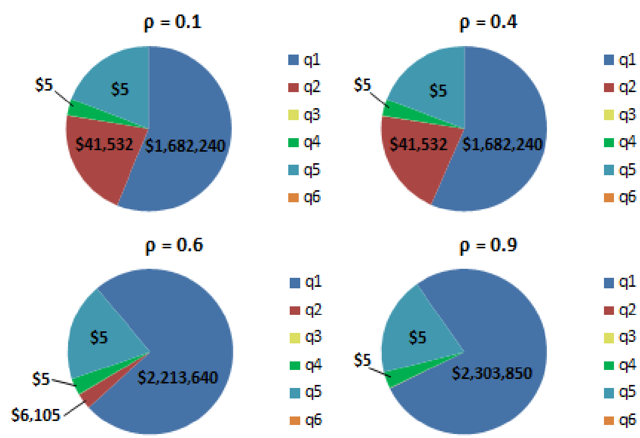
It is clear, though, that while the smaller program has the same expected payout, it is also less attractive to both types of consumers. For subjective probability, the consumer prefers a larger quantity allocated toward the jackpot. For indivisible goods, the consumer prefers a higher quantity of indivisible goods. As a result, we can conclude that there are returns to scale to having a larger PLS program.

5.3.2 Number of Prizes

I now allow the number of prizes to vary while holding the size of the prize pool fixed. I maintain the assumption that all but one prize is a gain, and keep the probability of each existing prize constant. An additional prize thus creates more winning tickets, while one less prize creates fewer winning tickets. As a result, the available prize pool that can be allocated to the top prize for subjective probability, or to prizes that allow indivisible goods consumption for indivisible goods, expands with one less prize, and contracts with an additional prize.

Under a 4-prize prize structure (see appendix), I re-solve for the optimal PLS structure and get the same qualitative result (see appendix). Since it is not necessary to allocate a portion of the prize pool to q_4, the prize pool expands for the remaining three prizes. Using a 6-prize prize structure (see appendix), I solve again for the optimal PLS structure with the first fives prizes as gains and q_6 as a loss:

Figure 5.2



With q_5 as a gain, the prize pool for the remaining prizes contracts. When $\rho = 0.1$ and $\rho = 0.4$, q_3 no longer allows the car to be consumed. As a result, there is a lower quantity, but higher quality, of indivisible goods tickets. When $\rho = 0.6$ and $\rho = 0.9$, the additional prize decreases the allocation to q_1, creating less of a "winner take all" system.

The optimal prizes structures in Section 4 are robust to the addition or subtraction of prizes in the sense that the qualitative results and intuition behind each theory do not change. Allowing the number of prizes to vary does, however, have an impact on the number of winning tickets and size of the available prize pool for each prize. As a result, the number of prizes affects the size of the "winner take all" system for subjective probability and the trade-off between quantity and quality for indivisible goods.

5.3.3 Prize Probabilities

Earlier, I solved for the optimal prize sizes given exogenously assigned prize probabilities. Using the prize sizes from the combined model optimal PLS design results in Figure 4.1, I solve the same maximization problem for the prize probabilities given these exogenously assigned prize sizes. With this specification, the maximization becomes:

$$\text{Maximize}_{p1,p2,p3,p4,p5}\ E(U_{cm}) = \rho\ E(U_{sp}) + (1\text{-}\ \rho)\ E(V_{ig})$$

subject to:

$$q_1 * p_1 + q_2 * p_2 + q_3 * p_3 + q_4 * p_4 + q_5 * p_5 = r_f * g$$

$$0 \leq p_1 \leq 1,\ 0 \leq p_2 \leq 1,\ 0 \leq p_3 \leq 1,\ 0 \leq p_4 \leq 1,\ 0 \leq p_5 \leq 1,\ 0 \leq p_6 \leq 1$$

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$$

If this maximization results in probabilities that are qualitatively similar to those calibrated for the model, this would suggest that the calibrated probabilities may be optimal. I define qualitatively similar here as a probability structure where the probabilities are progressively larger in size from p_1 to p_5. Solving the maximization problem, we arrive at the following probabilities for the world scenarios from Section 4:

Table 5.5

	p_1	p_2	p_3	p_4	p_5
ρ = 0	0	0	1/1,230.92	0	1/2.23
ρ = 0.1	0	0	1/1,230.92	0	1/2.23
ρ = 0.4	0	1/1,596.38	1/5,375.42	0	1/2.61
ρ = 0.6	1/1,168,216.11	1/2,370.70	0	0	1/2.42
ρ = 0.9	1/580,592.97	1/26,674.49	1/3,840.69	1/3,840.69	1/2.52
ρ = 1	1/580,592.97	1/26,674.49	1/3,840.69	1/3,840.69	1/2.52

When subjective probability theory is strongly preferred (ρ=1 and ρ=0.9), the probabilities are qualitatively similar to the assumed probabilities from the model. The probability for the largest prize, p_1, is the smallest followed by the remaining probabilities. The fact that the prize structure allocates the majority of the prize pool toward q_1 informs us that the subjective probability agent still prefers a "winner take all" prize structure. Given that the probabilities are not exactly the same, however, suggests that there may be alternative probabilities that make the PLS structure more attractive.

In the three scenarios where indivisible goods theory is preferred, the optimal probabilities for the prize structure are qualitatively different from the assumed probabilities in the model. The probabilities of prizes allowing a higher quality of indivisible goods consumption are 0. The entire prize pool, instead, is allocated to give the highest chance possible to consume the car.[7] Each prize structure pays out 476 winning tickets rather than 405. The kink in the indirect utility function at the car is large enough that the consumer values quantity over quality. That fact that the solved optimal probabilities for indivisible goods are qualitatively different from the calibrated probabilities in Section 4 suggests that the original probabilities were not optimal.

5.4 Discussion

This section demonstrates that the qualitative results from Section 4 are largely robust to alternative calibrations of the parameters. The qualitative results are not robust, however, to alternative probabilities of the prizes for indivisible goods. Given that I solve a constrained optimization rather than an unconstrained optimization, the results for the optimal PLS structure may be different when the prizes are not assigned exogenously. Unfortunately, I cannot relax the assumption of the exogenously assigned probabilities due to the computational restrictions of this thesis. I leave it for future research to continue this project and solve the unconstrained optimum for the number of prizes, prize sizes, and prize probabilities simultaneously.

[7] For $\rho = 0.4$, both q_2 and q_3 allow the car to be consumed. The sum of p_2 and p_3 is 1/1,230.92, the highest possible probability under the expected payout constraint.

6 Conclusion

The success of PLS programs internationally in encouraging savings behavior among those who did not save previously suggests such programs could be successful domestically as well. Although PLS programs are still outlawed throughout the majority of the U.S., there has been a movement over the past several years to legalize this alternative savings mechanism. Since 2010, six additional states have joined Michigan and passed legislation that legalizes of PLS programs. Several more states are in the process of doing so.[8]

Policy surrounding PLS programs should not only focus on their adoption, however, but also their design. This thesis explores the optimal design of PLS programs as a means to encourage higher savings behavior. Specifically, this thesis examines how two leading theories of gambling behavior, subjective probability and indivisible goods, inform the optimal design of PLS programs to maximize their attractiveness to lower income individuals. While it is beyond the scope of this thesis to determine the gambling behavior preferences in the U.S., my findings suggest that the optimal prize structure varies drastically depending on the assumed theory of gambling behavior. This thesis underscores the need for more empirical research to determine the gambling behavior preferences of the population given that the effectiveness of PLS programs hinges upon this question.

[8]Source: http://scorecard.assetsandopportunity.org/2012/measure/prize-linked-savings

7 Appendix

Calibration of prize probabilities and the number of prizes:

I based my exogenously chosen prize probabilities upon the $2 California Triple Win scratch ticket for two reasons. First, the $2 California scratch tickets have a qualitatively similar prize structure to the results I expected from my model. These tickets have one prize approximately large enough to purchase an indivisible good and probabilities that allow a large allocation toward a "winner take all" prize. Second, of the California scratch tickets, I selected the game with the most tickets under the assumption that quantity of tickets supplied is approximately equal to quantity demanded. High demand would indicate that the consumer finds the prize structure to be the most attractive. The $2 California Triple Win prize structure is presented below[9]:

Table 7.1

Prize ($)	**Probability**
20,000 (1)	1/600,000
1,000 (2)	1/40,000
200 (3)	1/12,000
100 (3)	1/1,690
40 (4)	1/414
20 (4)	1/100
10 (4)	1/50
5 (5)	1/26
4 (5)	1/13
2 (5)	1/13

While the California Triple Win has 10 prizes, the prize sizes are naturally distributed into 5 groups, indicated in parenthesis. This assumption follows from the idea that scratch ticket consumers have preferences for particular prizes sizes, and do not make a distinction between prizes of approximately the same size. I group the prizes

[9] Source: http://www.calottery.com/play/scratchers-games/$2-scratchers

such that the proportion of the largest prize to the smallest prize is minimized. In Section 5, I test the robustness of the calibrated 5-prize prize structure by allowing the number of prizes to vary.

To determine the probability of each of the 5 grouped prizes, I sum the probability of the prizes in each original group. For example, $p_4 = 1/414 + 1/100 + 1/50 = 1/30.84$. Section 5 tests the robustness of this calibration by solving for the optimal probabilities with given prize sizes.

Calibration of number of gains in subjective probability prize structure:

Given that scratch tickets predominantly pay out much larger prizes than the cost of the ticket, I assume that the smallest of the 5 prizes is a loss rather than a gain. I test the robustness of this assumption in Section 5 by changing the number of gains.

Calibration of r_f:

I calibrated $r_f = 0.31$ assuming the PLS program invests the deposits in 30-year treasuries, or another asset of similar risk. This reflects the behavior of banks in the U.S. with client deposits, and allows for a higher return on the program than a standard savings account. The current return on 30-year treasuries is approximately 3.1%.[10] This calibration may be conservative given that the 200-day moving average rate is 3.3%.

Calibration of g:

I calibrated the value of g = \$160 based upon the cost to participate in the Premium Bond program in the U.K (£100)[11] using a conversion rate of £1 = \$1.60.[12] As a successful large-scale program, this calibration demonstrates how a similarly sized

[10] Source: www.google.com/finance
[11] Source: Kearney et al. (2010)
[12] Source: www.google.com/finance

program can be designed more effectively. To test for the robustness of the calibration, I solve for a smaller sized program comparable to the existing U.S. programs in Section 5.

Calibration of n:

I calibrated n by determining the minimum number of participants necessary to guarantee that q_1 will pay out one ticket. This calibration makes the weakest assumption possible about the number of participants while still allowing all of the prizes to be paid out. Given that $q_1 = 1/600{,}000$, $n = 600{,}000$.

Calibration of indivisible goods:

The three indivisible goods I calibrated for my model are goods that low-income individuals are unable to consume due to bad access to credit markets. As shown in a study done by the Center for American Progress, low-income individuals are more likely to be denied for loans than higher-income individuals. The study cites that, without credit market access, few of these individuals can afford to purchase a car, home, or college education (Weller, 2007, p. 1).

In prioritizing the purchase of the three goods (car, home, then college education) I examined the Consumer Expenditure to find discrete jumps in the allocation of income to each good between income brackets. I worked under the assumption that a discrete jump in allocation toward a good at a lower income suggested higher prioritization of this good. The data show that a discrete jump in income allocated toward vehicle outlays (approximately 2%) occurs at a lower income than a discrete jump in owned dwellings (approximately 2%).[13] There is no discrete jump in the allocation of income toward education for the income brackets provided. As a result, the data from the Consumer

[13] Source: http://www.bls.gov/cex/csxstnd.htm

Expenditure Survey suggests that the car is prioritized in consumption, followed by the home and college education.

Calibration of y_0, z_0, and h_0:

Examining data compiled by a sample of lotteries, I found that median income before taxes of lottery consumers was approximately $35,000.[14] From here, I calibrated the effective tax rate for this income bracket using a tax rate calculator. With an effective tax rate of approximately 14%, I found that income after taxes is approximately y_0 = $30,000.[15] To calculate z_0, I used a living wage calculator for a sample of states in the U.S. Taking the median value, I found that living wage, or the minimum level of sustenance, is approximately z_0 = $23,000.[16] The amount of h_0 follows from the difference between income after taxes and the minimum level of sustenance. To account for error in the estimation of y_0 and z_0, I test for the robustness of my calibration in Section 5.

Calibration of $c(x_2)$ and $d(x_2)$:

The values of $c(x_2)$ are calibrated from several sources that analyze data on each of the three chosen indivisible goods. The price of a car, $13,105, is calibrated from the average price of new and used vehicle sales from RITA for 2011.[17] The price of a home,

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[14] Sources: http://www.winningwithnumbers.com/lottery/games/missouri/
http://www.kylottery.com/apps/customer_service/common_questions.html
http://www.state.wv.us/lottery/faq-complete.htm#Lottery4
http://www.molottery.com/learnaboutus/funfacts.shtm
http://www.naspl.org/index.cfm?fuseaction=content&menuid=14&pageid=1053
http://www.oregonlottery.org/About/Lottery101/MythsandFacts.aspx
http://retailers.coloradolottery.com/retailer-extranet/games

[15] Source: http://www.smartmoney.com/personal-finance/taxes/whats-your-average-tax-rate-9548/

[16] Source: http://www.livingwage.geog.psu.edu/

[17] Source:
http://www.bts.gov/publications/national_transportation_statistics/html/table_01_17.html

\$20,413, is calibrated from the average down payment on existing homes in 2011[18], as a percentage, multiplied by the average sales price of existing homes in 2011 from NAR.[19] The price of a college education, \$15,014 is taken from the NCES estimate for the average cost of a 4-year public institution.[20]

The utility derived from the indivisible goods, $d(x_2)$, is calibrated based upon the intuition that follows from the prioritization of the goods. The car, purchased first, provides the largest kink in the indirect utility function. The house and the college education provide smaller kinks, given that the agent chooses to consume these goods after the car. I test for the robustness of the calibrated parameter values in Section 5.

Calibration of δ and ε:

I calibrated $\delta = 0.6$ and $\varepsilon = 0.4$ such that divisible goods are favored more strongly than indivisible goods. Given that divisible goods include sustenance goods that are necessary for survival, it is intuitive that these goods are weighted higher than indivisible goods.

Solving the direct utility function for the indirect utility function:

This derivation is a qualitative exercise to demonstrate how the indirect utility function can be derived from the direct utility function. First, I maximize the direct utility function for x_1^* conditional upon different values of x_2. Given the choice of which indivisible goods are consumed, this solves for the optimal level of x_1. The maximization becomes:

[18] Source: http://marketing.lendingtree.com/pr/map_info_graphic.jpg
[19] Source: http://www.realtor.org/topics/existing-home-sales
[20] Source: http://nces.ed.gov/fastfacts/display.asp?id=76

$$\text{Maximize}_{x1,x2} U_{ig}(x_1,x_2) = x_1^{\delta} * (w(x_2))^{\gamma}$$

subject to:

$$y_1 \geq P_1 x_1 + c(x_2)$$

$$x_2 = \{0, 1, 2, 3\}$$

$$c(x_2) = \begin{cases} \text{if } x_2 = 1, & 13,105 \\ \text{if } x_2 = 2, & 33,518 \\ \text{if } x_2 = 3, & 48,532 \end{cases}$$

$$x_1 \geq 0$$

In this maximization, P_1 and $c(x_2)$ are the prices of x_1 and x_2, respectively. The first constraint specifies that total consumption is less than income in the second stage, while the last constraint specifies that consumption of divisible goods be greater than 0. Solving for each $x_1^*(x_2)$, I get the following maximized values for $U_{ig}(x_1,x_2)$ conditional upon the level of indivisible goods:

$$U_{ig}(x_1,x_2) = \begin{cases} \text{if } x_2 = 0, & (\frac{y_1}{P_1})^{\delta} \\ \text{if } x_2 = \{1, 2, 3\}, & (\frac{y_{1-P_2(x_2)}}{P_1})^{\delta}\ (w(x_2))^{\gamma} \end{cases}$$

By setting the conditional $U_{ig}(x_1,x_2)$ equal for adjacent levels of indivisible goods and solving for income, I calculate the kinks in income, $y_1(x_2)$, at which the consumer derives higher marginal utility from purchasing the next indivisible good. Calibrating $P_1 = 25$, the kinks in consumption are:

$$y_1(x_2) = \begin{cases} \text{if } x_2 = 1, & y_1 = \$36{,}105 \\ \text{if } x_2 = 2, & y_1 = \$56{,}518 \\ \text{if } x_2 = 3, & y_1 = \$71{,}532 \end{cases}$$

It is utility maximizing to purchase each indivisible good at $y_1(x_2) = z_0 + c(x_2)$. This follows from the intuition that once income exceeds the minimum level of sustenance allocated to x_1, the consumption of indivisible goods offers a higher marginal utility than divisible goods. While several of the parameters are calibrated to derive the same calibrated kinks demonstrated in the model, similar values for these parameters would provide the same qualitative results.

3 gains prize structure:

Table 7.2

3 Gains	**q_1**	**q_2**	**q_3**	**q_4**	**q_5**
Size ($)	2,972,250	116	5	0	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	99.87%	0.06%	0.07%	0%	0%

γ^+ = .56 prize structure:

Table 7.3

γ^+ = 0.56	**q_1**	**q_2**	**q_3**	**q_4**	**q_5**
Size ($)	2,877,070	30	5	5	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	96.67%	0.02%	0.07%	3.24%	0%

2-kink prize structure:

Table 7.4

2 Kinks	**q_1**	**q_2**	**q_3**	**q_4**	**q_5**
Size ($)	26,715	26,707	6,293	0	0
Tickets	1	15	405	19,449	115,385
% Prize Pool	0.90%	13.46%	85.64%	0%	0%

h_0 = $12,000 prize structure:

Table 7.5

h_0 = $12,000	**q_1**	**q_2**	**q_3**	**q_4**	**q_5**
Size ($)	36,587	36,562	1,224	14	14
Tickets	1	15	405	19,449	115,385
% Prize Pool	1.23%	18.42%	16.66%	9.19%	55.50%

h_0 = $2,000 prize structure:

Table 7.6

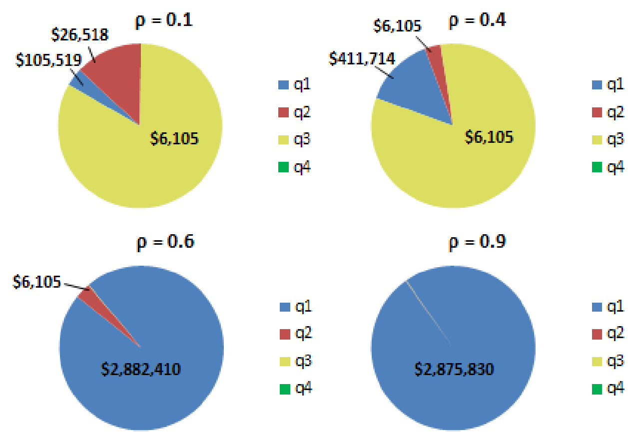
h_0 = $2,000	q_1	q_2	q_3	q_4	q_5
Size ($)	46,598	46,586	16	16	16
Tickets	1	15	405	19,449	115,385
% Prize Pool	1.57%	23.48%	0.22%	10.78%	63.95%

4-prize prize structure:

Table 7.7

Prize	Probability
q_1	p_1 = 1/600,000.00
q_2	p_2 = 1/40,000.00
q_3	p_3 = 1/1,481.40
q_4	p_4 = 1/30.84
0	p_5 = 1/1.03

Figure 7.1



6-prize prize structure:

Table 7.8

Prize	Probability
q_1	p_1 = 1/600,000.00
q_2	p_2 = 1/40,000.00
q_3	p_3 = 1/1,481.40
q_4	p_4 = 1/30.84
q_5	p_5 = 1/5.20
q_6	p_6 = 1/3.00
0	p_7 = 1/2.27

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