Problem Set 3 for Math 30

Date Due: Feb. 17, 2011

February 11, 2011

Sections 9.2-9.4
Chapter 9 Problems: 3, 6, 15, 17, 25, 26, 31, 32, 39, 42, 46, 51, 52

Additional:
Let $X_1, X_2, ..., X_n$ be a random sample from a $N(\mu, \sigma)$ distribution, where $\mu$ and $\sigma$ are both unknown, but are constant (and exist). Recall that $X = (n-1)s^2/\sigma^2$ has a chi-squared $(n-1)$ distribution. The steps below will lead you to prove consistency of $s^2$ for $\sigma^2$ directly (without using the trick we did in class).

a. What is the distribution of $X$ in terms of the Gamma distribution?
b. Let $Y = \frac{n-1}{n}s^2$. What is the distribution of $Y$?
c. Would you be able to use $Y$ as a pivot for $\sigma^2$? Explain in one sentence.
d. Find $Var(s^2)$ using the distribution you found in b.
e. Show consistency of $s^2$ for $\sigma^2$ without using the trick we did breaking up the summation.

Notes: 31 and 32 are practice with law of large numbers. 51 and 52 use a "trick" to show the sufficiency.
Other notes: Hardest problems on the homework are: 26, 39 (it says to show sufficiency the long way), 51, and 52 (if you don’t get the trick for the last two).