Amherst College
Economics 58
Fall 2010

Solutions to PS # 4

1. a. Use of the Lagrangian technique yields

$$y/(x+1) = p_x/p_y \quad \text{or} \quad p_y y = p_x x + p_x$$

Substitution into the budget constraint provides

$$x = \frac{I - p_x}{2p_x} \qquad y = \frac{I + p_x}{2p_y} \ .$$

Hence, changes in p_y do not affect x, but changes in p_x do affect y.

b. The indirect utility function is $V = \dfrac{(I + p_x)^2}{4p_x p_y}$

and this yields an expenditure function of

$$E = \sqrt{V4p_x p_y} - p_x$$

c. Clearly the compensated demand function for x depends on p_y , whereas the uncompensated function did not. By Shepherd's Lemma:

$$x^c = \frac{\partial E}{\partial p_x} = V^{0.5} p_x^{-0.5} p_y^{0.5} - 1$$

2. In class we showed that if price rises from p_1 to αp_1 the resulting compensating variation is given by : $CV = p_1 x_1 (\alpha^{b+1} - 1)/(b+1)$. Here are a few simulated values for the case $\alpha = 1.1$:

b	CV (proportion of $p_1 x_1$)
0	0.1
-0.5	0.098
-1.5	0.093
-5	0.082
-10	0.064

3. By definition $\alpha_n = \frac{-\Delta q}{\Delta p}\frac{p}{q} = \frac{q^1}{p^* - p^1} \cdot \frac{p^1}{q^1}$ so

$$CS = .5q^1(p^* - p^1) = \frac{.5}{\alpha_n} \cdot p^1 q^1$$ as was to be shown.