Math 130

Homework 6 Solutions

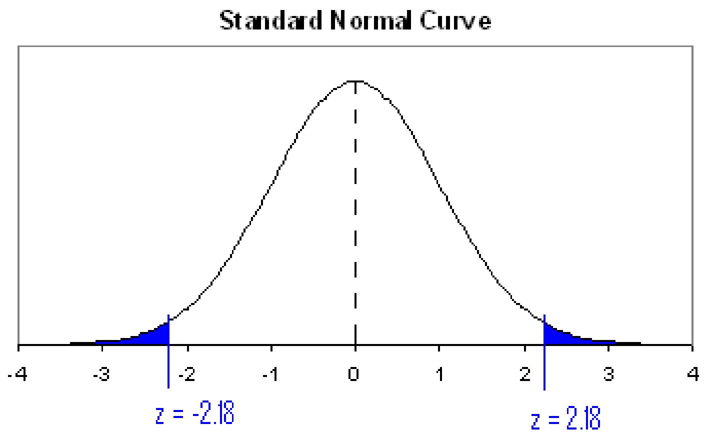
<u>Assignment</u>
Chapter 20: 12, ~~14~~, 16, 20, 24, 34
Chapter 21: 2, 8, 14, 16, 18
Chapter 22: 2, 8, 20, 39

Chapter 20

20.12] Got Milk?
The student made a number of mistakes here:

1. Null and alternative hypotheses should involve p, not $\hat{p}$.
2. The question asks if there is evidence that the 90% figure is not accurate, so a two-sided alternative hypothesis should be used. The alternative should be $H_A\!: \ p \neq 0.90$.
3. One of the conditions checked appears to be $n > 10$, which is not a condition for hypothesis tests. The Success/Failure Condition checks $np = (750)(0.90) \ = \ 675 \ > \ 10$ and $nq = (750)(0.10) \ = \ 75 \ > \ 10$. Also, the 10% condition is not verified.
4. $SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.90)(0.10)}{750}} = 0.01095$. The student used values of $\hat{p}$ rather than the null hypothesis value for p, here.
5. Value of Z is incorrect. The correct value is $Z = \frac{0.876-0.90}{0.011} = -2.18$
6. The p-value calculated is in the wrong direction. To test the given hypothesis, the lower tail probability should have been calculated.

   The correct, two-tailed P-value is $2P(Z < -2.18) = 2(0.0146) = 0.0292$.



7. The p-value is misinterpreted. Since the p-value is so low, there is moderately strong evidence that the proportion of adults who drink milk is different than the claimed 90%. In fact, our sample suggests that the proportion may be lower. There is only a 2.9% chance of observing a $\hat{p}$ as far from 0.90 as this simply from natural sampling variation.

~~20.14] Abnormalities.~~

~~a)~~ ~~Let p be the true percentage of children with genetic abnormalities. We're testing:~~

$$\cancel{H_0:\ p = 0.05}$$
$$\cancel{H_A:\ p > 0.05}$$

~~b)~~ ~~There is no reason to think that one child having genetic abnormalities would affect the probability that other children have them. These subjects are independent. This sample may not be random, but is probably representative of all children, with regards to genetic abnormalities. The sample of 384 children is less than 10% of all children. We have $np = (384)(0.05) = 19.2$ and $nq = (384)(0.95) = 364.8$ both greater than 10, so the sample is large enough.~~

~~c)~~ ~~The conditions have been satisfied, so a Normal model can be used to model the sampling distribution of the proportion, with $\mu_{\hat{p}} = p = 0.05$ and~~

$$\cancel{\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.05)(0.95)}{384}} = 0.0111.}$$

~~We can perform a one-proportion z-test. The observed proportion of children with genetic abnormalities is $\hat{p} = \frac{46}{384} = 0.1198$.~~

~~The value of Z is $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.1198-0.05}{\sqrt{\frac{(0.05)(0.95)}{384}}} = \frac{0.1198-0.05}{0.0111} = 6.28$.~~

~~This Z-value is way off to the right of the normal curve, so the p-value is essentially zero.~~

~~d)~~ ~~If 5% of children have genetic abnormalities, the chance of observing 46 children with genetic abnormalities in a random sample of 384 children is essentially 0.~~

~~e)~~ ~~With a p-value this low, we reject the null hypothesis. There is strong evidence that more than 5% of children have genetic abnormalities.~~

~~f)~~ ~~We don't know that environmental chemicals cause genetic abnormalities. We merely have evidence that suggests that a greater percentage of children are diagnosed with genetic abnormalities now, compared to the 1980s.~~

20.16] Educated Mothers.

a) Let p be the proportion of student's mothers who have not graduated from college in 2000. We wish to test:

$$H_0:\ p = 0.31$$
$$H_A:\ p \neq 0.31$$

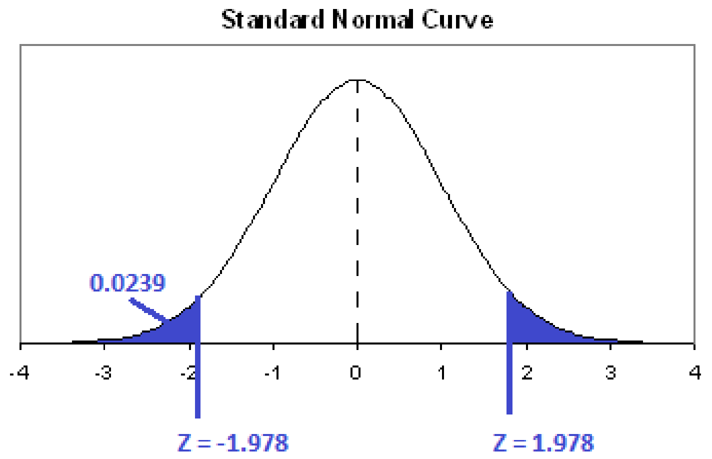
It's hard to tell by reading the direction of the alternative should be. I used the two-sided alternative because the problem asked about a change, but without any specific direction.

b) **Independence assumption:** It is reasonable to think that the students' responses are independent of one another.
Randomization condition: Although not specifically stated, we can assume that the National Center for Educational Statistics used random sampling.
10% condition: The 8368 students are less than 10% of all students.
Success/Failure condition: np= (8368)(0.31) = 2594.08 and nq= (8368)(0.69) = 5773.92 are both greater than 10, so the sample is large enough.

c) Since the conditions for inference are met, a Normal model can be used to model the sampling distribution of $\hat{p}$, with $\mu_{\hat{p}} = p = 0.31$ and

$$\sigma(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.31)(0.69)}{8368}} = 0.0051$$

The test statistic is $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.32-0.31}{\sqrt{\frac{(0.31)(0.69)}{8368}}} = 1.978$



The p-value is 2(0.0239) = 0.0478.

.d) With a p-value of 0.0478, we reject the null hypothesis, but just barely. There is some evidence to suggest that the percentage of students whose mothers are college graduates has changed since 1996. In fact, the evidence suggests that the percentage has increased.

e) This result is not meaningful. A difference this small, although statistically significant, is of little practical significance.

20.20] Satisfaction.

a) There is no reason to believe that one randomly selected customer's response will affect another's, with regards to complaints. The subjects can be assumed to be independent. The survey used 350 randomly selected customers. We've sampled less than 10% of the population: 350 customers are less than 10% of all possible customers. We have $n\hat{p} = 10 \geq 10$ and $n\hat{q} = 340 \geq 10$, so the sample is large enough. Since the conditions are met, we can use a one-proportion *z*-interval to estimate the proportion of the customers who have complaints. We have $\hat{p} = \frac{10}{350} 0.02857$.

$$\hat{p} \pm Z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.02857 \pm 1.96 \sqrt{\frac{(0.02857)(0.97143)}{350}} = 0.02857 \pm 0.01745$$
$$= (0.111\,, 0.0460)$$

We are 95% confident that between 1.1% and 4.6% of customers have complaints.

b) Let *p* be the true proportion of customers who have complaints. We are testing:

$$H_0\!:\ p = 0.05$$
$$H_A\!:\ p < 0.05$$

Since 5% is not in the 95% confidence interval, we will reject the null hypothesis. There is strong evidence that less than 5% of customers have complaints. This is evidence that the company has met its goal.

We've done a little more than meets the eye here. Checking that 5% is not in the interval is technically testing the two-sided alternative $H_A\!:\ p \neq 0.05$. However, recall that the two-sided *p*-value of a test is just twice that of the one-sided test. If we reject the null hypothesis using a two-sided alternative hypothesis, then we'll certainly also reject the null hypothesis using a one-sided alternative.

20.24] Scratch and dent.

Let *p* be the true percentage of damaged washers and dryers. We're testing:

$$H_0\!:\ p = 0.02$$
$$H_A\!:\ p < 0.02$$

Before proceeding, we should check our assumptions. It is reasonable to think of these machines as independent, unless multiple machines are handled together. We've been told that we have a random sample of 60 machines. The sample of 60 machines less than 10% of all the produced machines. We have $np = (60)(0.02) = 1.2$ and $nq = (60)(0.98) = 59$. Our sample is not large enough! We should not proceed with a one-proportion *Z*-test.

20.34] TV ads.

Let p be the true percentage of people who know that the company manufactures printers. We're testing:

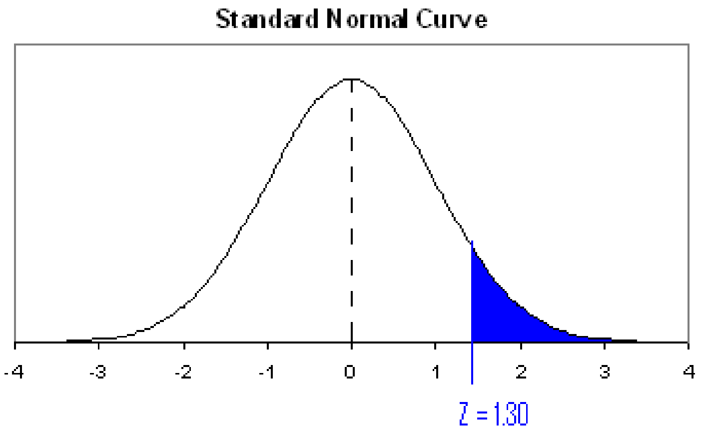
$$H_0\colon\ p = 0.40$$
$$H_A\colon\ p > 0.40$$

Our sample is independent. There is no reason to believe that the responses of randomly selected people would influence others. Our sample is random. The pollster contacted the 420 adults randomly. We've sampled less than 10% of the population: a sample of 420 adults is less than 10% of all adults. Finally, our sample is large enough. Both $np = (420)(0.40) = 168$ and $nq = (420)(0.60) = 252$ are greater than 10. We can proceed with the one-sample Z-test for a proportion.

The observed proportion of people who know the company manufactures printers is

$$\hat{p} = \frac{181}{420} = 0.4310.$$

The value of Z is $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.4310-0.40}{\sqrt{\frac{(0.40)(0.60)}{420}}} = \frac{0.4310-0.40}{0.0.0239} = 1.30.$



From the Z-table, the probability less than 1.30 is 0.9032.
The p-value is $P(Z > 1.30 | p = 0.40) = 1 - 0.9032 = 0.0968.$

Since the P-value = 0.0977 is fairly high, we fail to reject the null hypothesis. There is little evidence that more than 40% of the public recognizes the product. The company should conclude not to run commercials during the Super Bowl!

Chapter 21

21.2] Which alternative?

a) Two sided. Let *p* be the percentage of students who prefer plastic.

$$H_0\text{: } p = 0.50$$
$$H_A\text{: } p \neq 0.50$$

b) Two sided. Let *p* be the percentage of juniors planning to study abroad.

$$H_0\text{: } p = 0.10$$
$$H_A\text{: } p \neq 0.10$$

c) One sided. Let *p* be the percentage of people who experience relief.

$$H_0\text{: } p = 0.22$$
$$H_A\text{: } p > 0.22$$

d) One sided. Let *p* be the percentage of hard drives that pass all performance tests.

$$H_0\text{: } p = 0.60$$
$$H_A\text{: } p > 0.60$$

21.8] Significant again?

a) If 15.9% is the true percentage of children who did not attain the grade level standard, there is only a 2.3% chance of observing 15.1% of children (in a sample of 8,500) not attaining grade level by natural sampling variation alone.

b) Under old methods, 1,352 students would not be expected to read at grade level. With the new program, 1284 would not be expected to read at grade level. This is only a decrease of 68 students. The costs of switching to the new program might outweigh the potential benefit. It is also important to realize that this is only a potential benefit.

21.14] Spam.

$$H_0\text{: Message is real}$$
$$H_A\text{: Message is spam}$$

a) Type II. We failed to reject H_0 when it was false. The filter decided that the message was safe, when in fact it was spam.

b) Type I. We rejected H_0 when it was true. The filter decided that the message was spam, when in fact it was not.

c) This is analogous to lowering alpha. It takes more evidence to classify a message as spam.

d) The risk of Type I error is decreased and the risk of Type II error has increased.

21.16] More spam.

a) The power of the test is the ability of the filter to detect spam.

b) To increase the filter's power, lower the cutoff score.

c) If the cutoff score is lowered, a larger number of real messages would end up in the junk mailbox.

21.18] Alzheimer's.

a) H_0: The person is healthy
H_A: The person has Alzheimers

b) A Type I error is a false positive. It has been decided that the person has Alzheimer's disease when they don't.

c) A Type II error is a false negative. It has been decided that the person is healthy, when they actually have Alzheimer's disease.

d) A Type I error would require more testing, resulting in time and money lost. A Type II error would mean that the person did not receive the treatment they needed. A Type II error is much worse.

e) The power of this test is the ability of the test to detect patients with Alzheimer's disease. In this case, the power can be computed as $1-$ P(Type II error) $= 1- 0.08 = 0.92$.

Chapter 22

22.2] Science News. When the difference is not significant, it means that we did not have evidence to reject the null hypothesis. In this case, we do not have sufficient evidence to show that a higher proportion of people get their news from the internet than from television.

22.8] Graduation

a) **Randomization condition**: We can probably assume that the samples are representative of all recent graduates.
10% condition: Although large, the samples are less than 10% of all graduates.
Independent samples condition: The sample of men and the sample of women were drawn independently of each other.
Success/Failure condition: The samples are very large, certainly large enough for the methods of inference to be used.
$n_1\hat{p}_1 = 12{,}460(0.849) = 10{,}578.54 \geq 10$
$n_1\hat{q}_1 = 12{,}460(0.151) = 1{,}881.46 \geq 10$
$n_2\hat{p}_2 = 12{,}678(0.881) = 11{,}169.32 \geq 10$
$n_2\hat{q}_2 = 12{,}678(0.119) = 1{,}508.68 \geq 10$

It seems reasonable to conclude that the conditions have been satisfied. We will find a two-proportion z-interval.

b) Let p_m be the proportion of male high school graduates, and p_f be the proportion of female high school graduates.

The interval is:

$$(\hat{p}_m - \hat{p}_f) \pm Z^*\sqrt{\frac{\hat{p}_m\hat{q}_m}{n_m} + \frac{\hat{p}_f\hat{q}_f}{n_f}}$$
$$= (0.849 - 0.881) \pm 1.96\sqrt{\frac{0.849(0.151)}{12{,}460} + \frac{0.881(0.119)}{12{,}678}}$$
$$= -0.032 \pm 1.96(0.0043)$$
$$= -0.032 \pm 0.00844$$
$$= (-0.0404\,, -0.0236)$$

c) We are 95% confident that the proportion of 24-year-old American women who have graduated from high school is between 2.4% and 4.0% higher than the proportion of American men the same age who have graduated from high school.

d) The interval for the difference in proportions of high school graduates does not contain 0. Yes, there is strong evidence that women are more likely than men to complete high school.

22.20] Depression.

a) This was a prospective study, where people were recruited and then split into study groups based on their characteristics. We haven't discussed this design in class, so you are not responsible for this question.

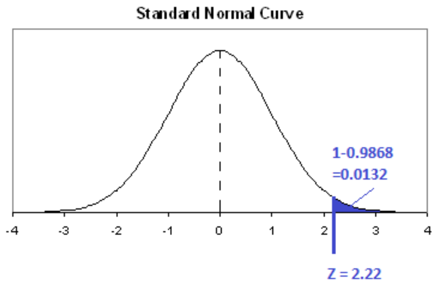
b) Let p_d represent the proportion of depressed patients who died of cardiac disease, and p_{nd} be the proportion of non-depressed patients who died of cardiac disease. We wish to test:

$$H_0:\ p_d = p_{nd}$$
$$H_A:\ p_d > p_{nd}$$

We have $\hat{p}_d = \frac{26}{89} = 0.2921$ and $\hat{p}_{nd} = \frac{67}{361} = 0.1856$. It certainly looks like depressed patients are more likely to die of cardiac disease, but we must do a hypothesis test to see if this difference is more than that of random variation from our sample.

c) **Randomization condition**: Assume that the cardiac patients followed by the study are representative of all cardiac patients.
10% condition: 361 and 89 are both less than 10% of all teens.
Independent samples condition: The groups are not associated.
Success/Failure condition:
$n_d\hat{p}_d = 89(0.2921) = 26 \geq 10$
$n_d\hat{q}_d = 89(0.7079) = 63 \geq 10$
$n_{nd}\hat{p}_{nd} = 361(0.1856) = 67 \geq 10$
$n_{nd}\hat{q}_{nd} = 361(0.8144) = 294 \geq 10$

The needed conditions do seem to be satisfied.



d) Our test statistic is

$$Z = \frac{(\hat{p}_d - \hat{p}_{nd}) - \Delta_0}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_d} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_{nd}}}}$$

Here, $\hat{p}_{pooled} = \frac{26+67}{89+361} = \frac{93}{450} = 0.2067$. This gives us

$$Z = \frac{(0.2921 - 0.1856) - 0}{\sqrt{\frac{0.2067(0.7933)}{89} + \frac{0.2067(0.7933)}{361}}} = \frac{0.1065}{0.0479} = 2.2234$$

The *p*-value is 0.0131, which is quite small (see graph above). We would reject the null hypothesis, and conclude that the proportion of depressed patients dying of cardiac disease is greater than that of non-depressed patients.

e) Assuming the proportion of cardiac death is the same for both groups, there is a 1.31% chance of getting the difference we did, or something greater.

f) We'd have made a type I error if our conclusion was actually incorrect.

22.39] Online Activity Checks.

Let p_{2004} represent the proportion of teens in 2004 who said their parents checked to see what web sites they visited, and p_{2006} be the same proportion for teens in 2006. We wish to test:

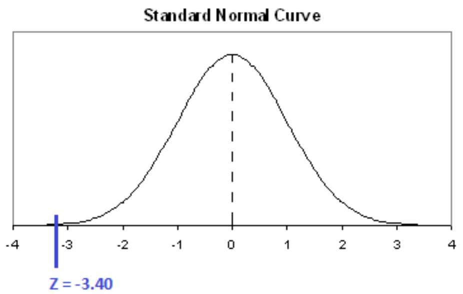
$$H_0:\ p_{2004} = p_{2006}$$
$$H_A:\ p_{2004} < p_{2006}$$

We have $\hat{p}_{2004} = 0.33$ and $\hat{p}_{2006} = 0.41$. It looks like more parents are checking on web sites, but we must do a hypothesis test to see if this difference is more than that of random variation from our sample.

We begin by checking the conditions:

Randomization condition: The samples were random.
10% condition: 811 and 868 are both less than 10% of all teens.
Independent samples condition: The samples were taken independently.
Success/Failure condition:

$$n_{2004}\hat{p}_{2004} = 868(0.33) = 286.44 \geq 10$$
$$n_{2004}\hat{q}_{2004} = 868(0.67) = 581.56 \geq 10$$
$$n_{2006}\hat{p}_{2006} = 811(0.41) = 332.51 \geq 10$$
$$n_{2006}\hat{q}_{2006} = 811(0.59) = 478.59 \geq 10$$



The conditions do seem to be satisfied.

Our test statistic is

$$Z = \frac{(\hat{p}_{2004} - \hat{p}_{2006}) - \Delta_0}{\sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_d} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_{nd}}}}$$

Here, $\hat{p}_{pooled} = \frac{286+332}{868+811} = \frac{618}{1679} = 0.368$. This gives us

$$Z = \frac{(0.33 - 0.41) - 0}{\sqrt{\frac{0.368(0.632)}{868} + \frac{0.368(0.632)}{811}}} = \frac{-0.08}{0.02355} = -3.40$$

The *p*-value is 0.0003, which is very small (see graph above). We would reject the null hypothesis, and conclude that the proportion of teens who reported parents checking their internet use has increased from 2004 to 2006.