A Note on Weighted Averages

I am quoting this from Taylor, An Introduction to Error Analysis (2nd. Ed.), (Sausalito: University Science Books, 1997), p.177.

> If $x_1, x_2, \ldots, x_N$ are measurements of a single quantity x, with known uncertainties $\sigma_1, \sigma_2, \ldots, \sigma_N$, then the best estimate for the true value of x is the *weighted average*
>
> $$x_{\text{wav}} = \frac{\sum w_i x_i}{\sum w_i}, \tag{1}$$
>
> where the sums are over all N measurements, $i = 1, \ldots, N$, and the weights w_i are the reciprocal squares of the corresponding uncertainties,
>
> $$w_i = \frac{1}{\sigma_i^2}. \tag{2}$$
>
> The uncertainty in x_{wav} is
>
> $$\sigma_{\text{wav}} = \frac{1}{\sqrt{\sum w_i}}, \tag{3}$$
>
> where, again, the sum runs over all of the measurements $i = 1, \ldots, N$.

As an example, consider two measurements $x_1 = 10(1)$ and $x_2 = 12(2)$. The uncertainties are $\sigma_1 = 1$ and $\sigma_2 = 2$, and the weights are $w_1 = 1$ and $w_2 = 0.25$. The weighted average is

$$x_{\text{wav}} = \frac{10 \cdot 1 + 12 \cdot 0.25}{1 + 0.25} = 10.4, \tag{4}$$

and the uncertainty in this weighted average is

$$\sigma_{\text{wav}} = \frac{1}{\sqrt{1 + 0.25}} \approx 0.89. \tag{5}$$

I would therefore quote the measured value as $x = 10.4(9)$.