Amherst College
Department of Mathematics and Statistics

Comprehensive Examination

◁ Analysis ▷

January 2018

Number: _____

Read This First:

- This is a closed-book examination. No books, notes, cell phones, electronic devices of any sort, or other aids are permitted. Cell phones are to be silenced and out of sight.

- Write your number (***not*** your name) in the above space.

- For any given problem, you may use the back of the ***previous*** page for scratch work. Put your final answers in the spaces provided.

- Additional sheets of paper will be available if you need them. If you use an additional sheet, label it carefully and be sure to include your number.

- In order to receive full credit on a problem, solution methods must be complete, logical and understandable. Show all your work, and justify your answers.

- The Analysis Exam consists of Questions 1–4 that total to 100 points.

For Department Use Only:

Grader #1: _____

Grader #2: _____

1. (a) [5 points] Let A be a nonempty subset of $\mathbf{R}$. State the definition of what it means for $b \in \mathbf{R}$ to be an upper bound for A.

   (b) [20 points] Suppose that A and B are nonempty subsets of the real numbers. Prove that if $\sup A < \sup B$, then there exists an element $b \in B$ that is an upper bound for A.

2. (a) [5 points] State the definition of a Cauchy sequence.

   (b) [5 points] State the definition of a bounded sequence.

   (c) [15 points] Prove that every Cauchy sequence is bounded.

3. (a) [5 points] Let $f \colon A \to \mathbf{R}$ be a function. Using the ϵ-δ definition, define what it means for f to be continuous at $c \in A$.

   (b) [20 points] Suppose that $f \colon A \to \mathbf{R}$ is continuous at $c \in A$. Prove using the above definition that $|f|$ is continuous at c.

4. (a) [5 points] State the Weierstrass M-test.

   (b) [20 points] Use part (a) to show that for any $r \in (0,1)$ the function $f(x) = \sum_{n=1}^{\infty} x^n$ is well-defined and continuous on $[-r, r]$.