There are 9 problems, for a total of 220 points. Please show all your work in the space provided or continue on the back of the pages. Please state clearly what result you are using at each step.

Please remember that all Math 12 exams are closed-book, closed-notes. Absolutely no books, notes, cheat sheets, calculators, or electronics (including cell phones) are allowed. Anything that has an on/off switch must be in the “off” position. The work you submit must be yours only. Amherst College Statement of Intellectual Responsibility applies to this examination as described in the course syllabus.

Good luck!

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1. Compute each of the following limits, or show that it does not exist.

(a) [10 points] \( \lim_{x \to +\infty} \frac{\cosh x}{e^x} \).

(b) [10 points] \( \lim_{x \to 0^+} \frac{\sqrt{x} \sin (2\sqrt{x}) - 2x + 4x^2}{x^3} \).
(c) [10 points] \( \lim_{n \to \infty} \frac{(-1)^n + \frac{1}{n}}{(-1)^n + n} \).

(d) [10 points] \( \lim_{n \to \infty} \frac{(n^2 + 1)^{\frac{1}{n}}}{n^{2n^2}} \).
2. Compute the following integrals:

(a) [10 points] \( \int \frac{(x + 1) \, dx}{x^4 + x^2} \).

(b) [10 points] \( \int \sqrt{16x^2 + 4^2} \, dx \).
3. Decide whether the following improper integrals converge or diverge.

Total for Question 3: 30

(a) [10 points] \[ \int_{2}^{+\infty} \frac{dx}{x + (\ln x)^2}. \]

(b) [10 points] \[ \int_{0}^{+\infty} \frac{dx}{|x - 1|^{3/4} + |x - 2|^{3/4}}. \]

(c) [10 points] \[ \int_{0}^{\frac{\pi}{2}} \cot 2x \, dx. \]
4. [15 points] Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = 4(x - 2)^2$ and $y = x^2 - 4x + 7$ about the $y$-axis. Sketch the region and a typical shell.
5. [15 points] Find the area of the region that lies inside the curve $r = 3\sin \theta$ and outside the curve $r = 2 - \sin \theta$. 
6. Decide whether the following series converge absolutely, converge conditionally, or diverge.

(a) [10 points] \[ \sum_{n=1}^{\infty} \left( \frac{(-1)^n n^{2n}}{(2n)^n} \right). \]

(b) [10 points] \[ \frac{1}{1} - \frac{1}{2} - \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} + \cdots. \]
(c) [10 points] \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n + \sqrt{n}}. \]

(d) [10 points] \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln^3 n}. \]
7. Consider the infinite series \( \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \).

(a) [10 points] Prove that this series converges and find its sum.

(b) [10 points] How many terms of the series should be added to evaluate the sum of the series correctly up to four decimal places?
8. Consider the series \( f(x) = \sum_{n=1}^{\infty} \frac{(2 - x)^n}{n^2} \).

(a) [10 points] Find all \( x \) for which this series converges. Indicate where the convergence is absolute or conditional.

(b) [10 points] Compute exactly \( f'(\frac{5}{2}) \).
9. Total for Question 9: 20

(a) [10 points] Find the terms up to the $x^5$ term in the following product of power series:

$$(1 + x + x^2 + x^3 + x^4 + x^5 + \cdots) \cdot (1 - x + x^2 - x^3 + x^4 - x^5 + \cdots).$$

What do you notice? Explain by finding a closed form for each of the series involved.

(b) [10 points] Find the first four terms (up to the $(x - \pi)^3$ term) of the Taylor expansion of $f(x) = \tan x$ around $a = \pi$. 
