1. [15 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \to \ln 3} \frac{1}{e^x - 3} - \frac{1}{3x - \ln 27}$  
(b) $\lim_{x \to \infty} \left( \cosh \left( \frac{1}{x} \right) - \frac{5}{x} \right)^x$

2. [30 Points] Evaluate each of the following integrals.

(a) $\int \frac{1}{(x^2 + 4)^{\frac{3}{2}}} \, dx$  
(b) $\int x \arcsin x \, dx$  
(c) $\int \frac{x^4 + x^3 + 4x^2 + 5x + 4}{x^3 + 4x} \, dx$

3. [20 Points] For each of the following improper integrals, determine whether it converges or diverges. If it converges, find its value.

(a) $\int_3^\infty \frac{1}{x^2 - 4x + 7} \, dx$  
(b) $\int_0^1 \frac{e^\pi}{x^2} \, dx$

4. [10 Points] Find the sum of each of the following series (which do converge):

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n+2}}{2^{2n-1}}$  
(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots$  
(c) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{9^n (2n)!}$

5. [25 Points] In each case determine whether the given series is absolutely convergent, conditionally convergent, or diverges. Justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n + 1}$  
(b) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2}$  
(c) $\sum_{n=1}^{\infty} \frac{1}{5^n + 1}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n (9 + n^2)}{n^7 + 1}$  
(e) $\sum_{n=1}^{\infty} \frac{(-1)^n \pi^n (3n)!}{n^n (27)^n (n!)^2}$
6. [10 Points] Find the Interval and Radius of Convergence for the power series \[
\sum_{n=0}^{\infty} \frac{(-1)^n (4x + 1)^n}{(n^2 + 1) 7^n}.
\] Analyze carefully and with full justification.

7. [5 Points] Write the MacLaurin Series for \( f(x) = x^5 \sin(x^3) \). Use this series to determine the eighth and ninth derivatives of \( f(x) \) at \( x = 0 \). (Do not compute out those derivatives manually.)

8. [10 Points] Please analyze with detail and justify carefully.
(a) Find the MacLaurin series representation for \( f(x) = xe^{-x^7} \). Your answer should be in sigma notation \( \sum_{n=0}^{\infty} \).

(b) Use the MacLaurin series representation for \( f(x) = xe^{-x^7} \) from Part(a) to
\[
\text{Estimate } \int_0^1 xe^{-x^7} \, dx \text{ with error less than } \frac{1}{10}.
\]
Justify in words that your error is indeed less than \( \frac{1}{10} \).

9. [15 Points] Consider the region bounded by \( y = 1, y = \ln x, x = 1 \). Rotate the region about the line \( x = -1 \). Compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

10. [15 Points] Parametric Curves
(a) Consider the Parametric Curve represented by \( x = \frac{t^3}{3} - \frac{e^{2t}}{2} \) and \( y = 2te^t - 2e^t \). Compute the arclength of this parametric curve for \( 0 \leq t \leq 1 \).

(b) Consider the Parametric Curve represented by \( x = \cos^3 t \) and \( y = \sin^3 t \). Compute the surface area obtained by rotating this curve about the \( y \)-axis, for \( 0 \leq t \leq \frac{\pi}{2} \).

11. [15 Points] Compute the area bounded outside the polar curve \( r = 2 + 2 \cos \theta \) and inside the polar curve \( r = 6 \cos \theta \). Sketch the Polar curves.

12. [10 Points] Find the general solution for each of the following differential equations.
\[\frac{dy}{dx} = \frac{e^y}{\sqrt{1 - x^2}}\]
\[x \frac{dy}{dx} = x^7 \cosh x + 4y\]