1. (5 points each) Compute the following limits. You should say if the limit is equal to a particular value, if it is $\pm \infty$, or if it does not exist.

(a) $\lim_{x \to 0} \frac{xe^x - \sin x}{1 - \cos(3x)}$.

(b) $\lim_{x \to 0} (e^x - 4x)^{1/x}$.

2. (8 points each) Compute the following integrals.

(a) $\int \frac{2x^3 - 4}{x^3 - 2x^2} \, dx$.

(b) $\int \sin \theta \tan^2 \theta \, d\theta$.

(c) $\int \frac{x}{\sqrt{x^2 + 2x + 5}} \, dx$.

3. (10 points) The curve $y = 2\sqrt{x}$, $1 \leq x \leq 3$, is revolved about the $x$-axis. Find the area of the resulting surface.

4. (8 points each) For each of the following improper integrals, determine whether the integral converges or diverges. If it converges, find its value.

(a) $\int_1^\infty \frac{dx}{x^2 + 3}$.

(b) $\int_0^1 \ln x \, dx$.

5. (5 points each) Compute the following sums.

(a) $\sum_{n=1}^{\infty} \frac{3 + 4^n}{5^n + 1}$.

(b) $1 - \ln 7 + \frac{(\ln 7)^2}{2!} - \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} - \cdots$.
6. (8 points each) Determine whether each series converges absolutely, converges conditionally, or diverges. Justify your answers.

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{7n+1} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 2^n} \)

(c) \( \sum_{n=1}^{\infty} \frac{(2n)!}{2^n (n!)^2} \)

7. (10 points) The region under the curve \( y = \sin x \), \( 0 \leq x \leq \pi/2 \), is revolved about the line \( x = 8 \). Find the volume of the resulting solid.

8. (10 points) Find the interval of convergence of the power series

\[ \sum_{n=1}^{\infty} \frac{(-3)^n (x + 1)^n}{\sqrt{n}}. \]

9. (8 points each) Let \( C \) be the curve given by the parametric equations

\[ x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 3. \]

(a) Find the equation of the line tangent to \( C \) at the point where \( t = 2 \).

(b) Find the length of \( C \).

10. (5 points each)

(a) Find the Maclaurin series for the function \( f(x) = x^2 \sin(x^2) \).

(b) Use your answer to part (a) to estimate \( \int_{0}^{1} x^2 \sin(x^2) \, dx \) with error less than 0.01. Be sure to justify the fact that the error in your approximation is less than 0.01.

11. (10 points) Sketch the curves given in polar coordinates by the equations \( r = 2 + 2 \cos \theta \) and \( r = 3 \), and find the area of the region that is inside the first curve and outside the second.