Math 211, Section 03, Fall 2014

**Final Exam, Monday, December 15, 2014**

*Instructions:* Do all twelve numbered problems. If you wish, you may also attempt the three optional bonus questions. **Show all work**, including scratch work. Little or no credit may be awarded, **even when your answer is correct**, if you fail to follow instructions for a problem or fail to **justify your answer.**
If your answer for a given problem is a sum of fractions with different denominators, you may leave it that way. Otherwise, simplify your answers whenever possible.
If you have time, check your answers.

**WRITE LEGIBLY. NO CALCULATORS.**

1. **(12 points)** A particle is travelling in such a way that its **velocity** vector at time $t$ is given by $\vec{r}\,'(t) = \langle t^2, \sqrt{2}t, 1\rangle$.

  **(1a)** How far does the particle travel from time $t = 1$ to time $t = 2$?

  **(1b)** What is the curvature of the path the particle traces out at the point it passes through at time $t = 2$?

2. **(18 points)** Let $f(x, y) = \begin{cases} \dfrac{2x^3 + y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

  **(2a)** Compute $f_x(0, 0)$ and $f_y(0, 0)$.

  **(2b)** Compute $D_{\vec{u}} f(0, 0)$, where $\vec{u}$ is the unit vector pointing in the direction of $\langle 1, -1\rangle$.

  **(2c)** Based on your answers to parts (a) and (b), explain (briefly) why $f$ cannot be differentiable at $(0, 0)$.

3. **(18 points)** Find and classify (as local minimum, local maximum, or saddle point) every critical point of the function $f(x, y) = x^2y - 3x^2 - 6y^2 + 2$.

4. **(12 points)** Find the point on the ellipse $x^2+6y^2+3xy = 40$ with the largest $x$-coordinate.

5. **(20 points)** Find the volume of the region that is inside the sphere $x^2 + y^2 + z^2 = 9$ and also inside the cylinder $x^2 + y^2 = 3x$.
(Note that the cylinder is centered around a vertical line that is *not* the $z$-axis.)

6. **(20 points)** Let $E$ be the solid lying
  - inside the sphere $x^2 + y^2 + z^2 = 9$,
  - outside the sphere $x^2 + y^2 + z^2 = 1$,
  - below the cone $z = \sqrt{x^2 + y^2}$, and
  - in the first octant.

Compute $\displaystyle\iiint_E y\,dV$.

7. **(20 points)** Let $E$ be the solid bounded by the surfaces $y = \sqrt{x}$, $x = 2y$, $z = 4$, and $x + z = 4$. Compute the volume of $E$.

8. **(15 points)** Let $C$ be the quarter of the circle $x^2 + y^2 = 9$ in the second quadrant, i.e., the quarter-circle arc from $(0, 3)$ to $(-3, 0)$. Compute $\displaystyle\int_C x^2y\,ds$.

9. **(15 points)** Let $C$ be the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$, oriented counterclockwise. Let $\vec{F}(x,y) = \langle 3y^2, x^2y + \cos^8 y\rangle$. Compute $\int_C \vec{F} \cdot d\vec{r}$.

10. **(15 points)** Let $C$ be the line segment from $(1,0,-1)$ to $(0,-2,2)$.
Compute $\int_C 2y\,dx - 3xy\,dy + z^2\,dz$.

11. **(15 points)** Let $\vec{F}(x,y) = \langle 2xy + 6x^2, x^2 - y^3\rangle$.

   **(a)** Show that $\vec{F}$ is conservative by finding a potential function $f(x,y)$ for $\vec{F}$.

   **(b)** Let $C$ be the curve parametrized by $\vec{r}(t) = \langle t(t-2), t^2(t-3)\rangle$, for $1 \le t \le 3$.
   Compute $\int_C \vec{F} \cdot d\vec{r}$.

12. **(20 points)** Let $\vec{F}(x,y,z) = \langle y^2, z, x\rangle$, and let $S$ be the part of the paraboloid $z = 3x^2 + 3y^2$ in the first octant and below the plane $z = 3$, oriented downward.
Compute $\iint_S \vec{F} \cdot d\vec{S}$.

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**OPTIONAL BONUS A. (2 points)** Let $C$ be the circle $(y-2)^2 + z^2 = 1$ in the $yz$-plane, and let $S$ be the (surface of the) torus formed by rotating $C$ around the $z$-axis. Compute the surface area of $S$.

**OPTIONAL BONUS B. (2 points)** Let $C$ be the portion of the circle $x^2 + y^2 = 1$ in the first quadrant, oriented counterclockwise, i.e., running from $(1,0)$ to $(0,1)$.
Let $\vec{F}(x,y) = \langle 2xy^3\cos(x^2), x^2 + 3y^2\sin(x^2)\rangle$. Compute $\int_C \vec{F} \cdot d\vec{r}$.

**OPTIONAL BONUS C. (1 point)** On December 6, 2014, there was a special runoff election to decide a Senate race for a certain US State. Which one of the 50 states was it?