

1. Evaluate the following limits. Be sure to justify your work.

(a) [6 points] $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 + 2x + 2}$

(b) [6 points] $\lim_{t \rightarrow 3} \frac{\sqrt{1+t} - 2}{t - 3}$

(c) [6 points] $\lim_{x \rightarrow -\infty} \frac{1 + x^3}{2 - x^2}$

(d) [6 points] $\lim_{x \rightarrow 2^+} \frac{x^2 + 2x - 10}{x - 2}$

(e) [6 points] $\lim_{x \rightarrow 1} \frac{\frac{x+1}{x+3} - \frac{1}{x+1}}{x^2 - 1}$

2. Compute the following derivatives.

(a) [6 points] $\frac{d}{dx} \left(\frac{x^2 + \pi^2}{x^3 + \sqrt{2}^3} \right)$. Do *not* simplify your answer.

(b) [6 points] $\frac{d}{dT} \sqrt{\frac{kT}{M^3}}$ for constants k and M . Do *not* simplify your answer.

(c) [9 points] $\left((x^2 + 1)^3 (3x - 1)^2 \right)'$. Please simplify your answer fully.

(d) [9 points] $\frac{d^2}{dx^2} \left(\frac{x^2}{x-1} \right)$. Please simplify your answer fully.

3. [10 points] Find the equation of the line tangent to the curve $y = x\sqrt{(x-2)^3}$ at the point where $x = 3$.

4. [10 points] Let $f(x) = \frac{x}{x+2}$. Compute $f'(x)$ using the limit definition of derivative.

5. [10 points] Find the absolute minimum and maximum values of the function $f(x) = x(x^2 - 7)^3$ on the interval $[0, 2]$. Also tell me where these occur.

6. [20 points] Draw the graph of the curve $y = x + \frac{4}{x}$. Please indicate where the function is increasing and decreasing and where the graph is concave up and concave down. Also label local and absolute maxs and mins (if any) and points of inflection (if any), and horizontal and vertical asymptotes (if any).

7. [20 points] Two cars start driving from the same location at 10 am. One car drives north at 30 mph and the other car drives east at 40 mph. How fast is the distance between the cars changing at 11 am?

8. [20 points] Find two positive numbers with product 8 such that the sum of the first and twice the second is as small as possible. Give a careful solution and be sure to explain how you found the interval.

9. Consider the following graph, which represents the distance d run at time t during a race:

- (a) [4 points] What was the average speed of the runner between $t = 0$ and $t = 4$?
- (b) [6 points] At approximately what time was the runner running the fastest? I want you to give me one specific time, not an interval. Explain your reasoning using concepts from the course.

10. The function $f(x) = \frac{\sqrt{x-1}}{x}$ is defined on the interval $[1, \infty)$.

- (a) [8 points] Compute $f'(x)$ and simplify your answer.
- (b) [4 points] $f(x)$ has a unique critical number in the interval $(1, \infty)$. Find it.
- (c) [4 points] Determine whether the critical number found in part (b) is a local minimum, local maximum, or neither.

11. Consider the function

$$g(x) = \begin{cases} 2 - x^2 & x < 0 \\ 3 & x = 0 \\ |x - 2| & x > 0. \end{cases}$$

- (a) [12 points] Draw the graph of $g(x)$.
- (b) [6 points] Find all numbers a such that $g(x)$ is not continuous at $x = a$. For each a , use the definition of continuity to explain why continuity fails.
- (c) [6 points] Find all numbers a such that $g(x)$ is not differentiable at $x = a$. For each a , explain why differentiability fails.