Math 211, Section 01, Fall 2017

Final Exam, Friday, December 22, 2017

Instructions: Do all twelve numbered problems. If you wish, you may also attempt the three optional bonus questions. **Show all work**, including scratch work. Little or no credit may be awarded, **even when your answer is correct**, if you fail to follow instructions for a problem or fail to **justify your answer.** If your answer for a given problem is a sum of fractions with different denominators, you may leave it that way. Otherwise, simplify your answers whenever possible. If you need more space, use the back of any page. If you have time, check your answers.

WRITE LEGIBLY. NO CALCULATORS.

1. **(11 points)** Find an equation for the plane that contains the line $\vec{r}(t) = \langle 2+t, 3t, 1-t\rangle$ and the point $(2, 3, -1)$.

2. **(12 points)** Let $f(x, y) = 2 + \ln(3x + y^2)$. Write the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(-1, 2, 2)$, and then use it to estimate $f(-1.2, 2.1)$.

3. **(17 points)** Let $f(x,y) = \begin{cases} \dfrac{2x^3 + 4xy - y^3}{3x^2 + y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

   3a. Prove that f is not continuous at $(0, 0)$.

   3b. Compute $f_x(0, 0)$ and $f_y(0, 0)$.

4. **(20 points)** Find and classify (as local minimum, local maximum, or saddle point) every critical point of the function $f(x, y) = 6x^2 + 3y^2 - xy^2 + 9$.

5. **(15 points)** Find the maximum and minimum values of the function

$$f(x, y) = x^2 + 8y$$

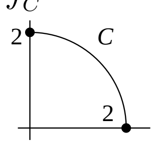
on the ellipse $x^2 + 2y^2 = 18$.

6. **(20 points)** Let E be the solid bounded below by the surface $z = y^2$, bounded above by the plane $z = 1$, and bounded in the back and front by the planes $y = x$ and $x = 2$, respectively. Suppose that the density of E is given by $\rho(x, y, z) = 10z$. Compute the mass of E.

7. **(15 points)** Let E be the solid lying
   - inside the sphere $x^2 + y^2 + z^2 = 1$,
   - above (i.e., inside) the cone $z = \sqrt{x^2 + y^2}$, and
   - inside the first octant.

Compute $\displaystyle\iiint_E 8z\, dV$.

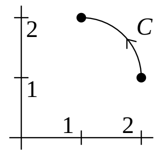
8. **(15 points)** Let C be the quarter-circle arc of radius 2 in the first quadrant of the xy-plane, as shown in the figure. Compute $\displaystyle\int_C x^3\, ds$.



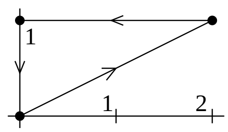
9. **(15 points)** Let $\vec{F}(x,y) = \left\langle 6x - 3x^2y, 6y^2 - x^3 \right\rangle$.

9a. Show that $\vec{F}$ is conservative by finding a potential function for $\vec{F}$.

9b. Let C be the quarter-circle path running from $(2,1)$ to $(1,2)$, counterclockwise along the arc of the circle $(x-1)^2 + (y-1)^2 = 1$, as shown below. Compute $\int_C \vec{F} \cdot d\vec{r}$.



10. **(20 points)** Let C be the boundary of the triangle with vertices $(0,0)$, $(2,1)$, and $(0,1)$, oriented counterclockwise, as shown in the figure. Let $\vec{G}(x,y) = \langle xy^2, 3x^2y + \cos^8 y \rangle$. Compute $\int_C \vec{G} \cdot d\vec{r}$.



11. **(20 points)** Let S be the closed surface consisting of

- the portion of the paraboloid $z = 2 - 2x^2 - 2y^2$ above the xy-plane, and
- the disk $x^2 + y^2 \leq 1$ in the xy-plane,

oriented outward. Let $\vec{G}(x,y,z) = \langle 2x^3, y^2z, -yz^2 \rangle$. Use the Divergence Theorem to compute the flux $\iint_S \vec{G} \cdot d\vec{S}$ of $\vec{G}$ through S.

12. **(20 points)** Let S be the portion of the surface $z = x^2 + y^2$ that lies in the first octant, and below the plane $z = 4$, oriented downward. Let $\vec{F}(x,y,z) = \langle y, -x, xz \rangle$. Compute the flux $\iint_S \vec{F} \cdot d\vec{S}$ of $\vec{F}$ through S.

OPTIONAL BONUS A. (2 points) Let C be the portion of the graph of $y = \sin x$ from the point $(0,0)$ to the point $(\pi, 0)$. Compute $\int_C (9x^2y^2 + y)\,dx + (6x^3y - \sin y)\,dy$.

OPTIONAL BONUS B. (2 points) Find a vector field $\vec{F}(x,y,z)$ such that

$$\operatorname{curl}(\vec{F}) = \left\langle 3yz^2 - xy, 4xyz - 3x^3, yz - 2xz^2 \right\rangle.$$

OPTIONAL BONUS C. (1 point) In the past week, it has been reported that the US Department of Health and Human Services is prohibiting officials at the Centers for Disease Control and Prevention from using seven terms in official documents. List three of those seven terms.