**Math 211 Fall 2017 Final Exam** **December 22, 2017**

You are not allowed to use books, notes, calculators, or other aids. Explain your answers completely and clearly to receive full credit. Please turn off all electronic devices and anything else that could cause distractions. There are 8 problems with a total of 100 points.

1. (12 points) Consider the function

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^3 + y^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Compute $f_x(0,0)$ and $f_y(0,0)$.

   (b) Is $f(x,y)$ continuous at $(0,0)$? Explain.

2. (12 points) Consider the function $f(x, y) = y \sin(\pi x y)$.

(a) In what direction does $f$ increase most rapidly if you are at the point $(1, -1)$? Express as a unit vector.

(b) In what direction does $f$ decrease most rapidly if you are at the point $(1, -1)$? Express as a unit vector.

(c) In what directions does the directional derivative of $f$ equal 0 if you are at the point $(1, -1)$? Express as unit vectors.

(d) Find the directional derivative $D_{\mathbf{u}} f(1, -1)$ in the direction $\mathbf{u} = \langle 1, 0 \rangle$.

3. (12 points) Let $f(x, y) = \ln(2x+y)$. Write the equation of the tangent plane to the surface $z = f(x, y)$ at the point (-1,3,0) and then use it to estimate $f(-1.1, 2.9)$.

4. (12 points) Find the critical points of the function

$$f(x, y) = 2x^3 - y^2 + 6xy$$

and determine whether each critical point is a local maximum, local minimum, or saddle point.

5. (12 points) Find the absolute maximum and minimum values of the function

$$f(x, y) = x + 2y$$

subject to the constraint $x^2 + y^2 \leq 4$.

6. (16 points) The region $E$ lies above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 2$, whose density is given by the function $f(x, y, z) = 10 - z$. **Set up but do not evaluate** the appropriate integral to compute the mass of this region in

(a) cylindrical coordinates.

(b) spherical coordinates.

7. (12 points) Let $\vec{F}(x, y) = \langle\, 2xy + 6x^2,\ x^2 - y^3 \,\rangle$.

(a) Show that $\vec{F}$ is conservative by finding a potential function $f(x, y)$ for $\vec{F}$.

(b) Let $C$ be the curve parametrized by $\vec{r}(t) = \langle\, t^2 - 2t,\ t^3 - 3t \,\rangle$, for $1 \leq t \leq 2$. Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.

8. (12 points) Let $C$ be the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$, oriented counterclockwise. Let $\vec{F}(x,y) = \langle\, 3y^2,\ x^2y + \cos^8 y\,\rangle$. Compute $\int_C \vec{F}\cdot d\vec{r}$.

**Bonus problem** (5 points) Find all points at which the ellipsoid $x^2 + 2y^2 + 2z^2 = 5$ is tangent to the hyperboloid $x^2 + y^2 - (z + 1)^2 = 2$.