Department of Mathematics and Statistics
Mellon Grant Proposal
November 25, 2017

We would like to apply for a Mellon Grant to support a deep examination of our calculus sequence, which impacts a large proportion of Amherst students. In recent years, around 75% of graduating seniors have taken at least one mathematics or statistics course, and most of these students begin somewhere in our three-semester calculus sequence (MATH 111, 121, 211). These courses have been tremendously successful in attracting and retaining students in mathematics. We want to build on this success while increasing our support for students who go on to major in Mathematics as well as those who major in other disciplines. The focus of this proposal is on (1) expanding the possibilities for pedagogical approaches in our calculus courses by reducing specified content that must be covered; (2) articulating higher level learning goals as the main driver determining what should be covered in calculus courses; and (3) learning about and then adopting appropriate pedagogical strategies that maximize learning for all students and that best suit each instructor. The central goal is to help our students build and retain essential skills in calculus courses and successfully transfer these skills to other disciplines and to more advanced mathematics courses. An emphasis on conceptual understanding will help all students as they apply calculus at higher levels, whether in mathematics or outside it. These changes are motivated by comments from an external review of our department, by a general desire for opportunities to think deeply on issues of teaching and learning, and by a desire on the part of some faculty to incorporate new pedagogical approaches into their classes.

The current context

MATH 111, 121 and 211 are designed as a three-semester sequence, though a significant proportion of students are placed into either MATH 121 or 211 depending on their high school background in calculus:

- **MATH 111: Introduction to Calculus**: Basic calculus of functions of a single variable; derivatives; integrals and the Fundamental Theorem of Calculus
- **MATH 121: Intermediate Calculus**: Further integration techniques; sequences and series; applications of integration
- **MATH 211: Multivariable Calculus**: Calculus of vector-valued functions, and functions of more than one variable

We teach 5-7 sections of each of these classes every academic year (plus versions of MATH 111 that provide additional support), for a total of around 20 sections annually with typically 24-35 students per section. Each faculty member teaches their section(s) independently, though all the faculty teaching sections of the same course meet occasionally during the semester to compare notes. Faculty teaching these classes are expected to cover a specified and long list of topics, and the textbooks are chosen by the Department to ensure some continuity for students from one semester to the next. While maintaining good consistency across instructors, this situation also results in a rigidity that prevents adoption of pedagogies other

than traditional lectures. In addition, the currently written learning goals for calculus are very concrete and content-oriented (though we do informally have unwritten higher goals in mind).

These courses are largely focused on developing the underlying theory of calculus, with attention to topics needed in subsequent mathematics and statistics classes. The majority of our faculty teach these classes in a lecture-based format with frequent homework assignments that give students practice at using the concepts and techniques introduced in lecture. In some sections, time in class is occasionally spent on group work for supervised practice. The assignments and exams consist mainly of short-answer problems that involve particular types of calculation. Students are expected to have practiced such calculations on homework assignments so that on exams they are able to apply the appropriate method quickly and accurately.

In addition to the mathematical content, our classes focus on developing study habits that will benefit students as they progress to higher-level mathematics and statistics classes. These habits include making good use of office hours, paying attention to detail, and using mathematical notation and terminology accurately when solving problems. Clear communication of mathematical answers is also stressed, to prepare students for the transition to more advanced courses in which they will learn to write mathematical proofs. However, writing out complete arguments to problems involving more than a direct calculation is not consistently taught across the sections and the textbooks do not support building these skills. We have observed some students struggling in more advanced courses with proof-writing, suggesting we need to do more in calculus to set a foundation for these skills.

Many students in our calculus courses go on to take courses in other departments that require calculus (such as chemistry, economics, and physics) and/or higher-level mathematics and statistics courses. These students need to learn how to transfer knowledge and skills gained in calculus to their next courses. In general, our calculus courses are pivotal for our students and must meet a broad array of content and skills goals aimed at preparing students to succeed in their further quantitative courses at the college.

Proposed changes and their rationale

In 2012, an external review committee advised our Department "to begin a serious and intentional conversation about the curriculum," and recommended specifically that, as we explore modifications to our program, we take note of a Curriculum Guide[1] published by the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA). In its discussion of the calculus curriculum, this guide (updated in 2015) says (CUPM p.18):

> The emphasis within Calculus has traditionally been on derivatives as slopes of tangent lines and integrals as areas—a very static interpretation that makes it difficult for many students to transfer these tools to dynamical situations.

We want our students to be able to use knowledge obtained in calculus courses in a variety of situations, whether in courses for other departments or in the projects they work on after

[1] *Committee on the Undergraduate Program in Mathematics (CUPM) Curriculum Guide*, Mathematical Association of America, 2015.

leaving Amherst. Revamping the calculus syllabi will enable all instructors to spend more time on teaching in a way that encourages long-term learning. At the same time, our strong departmental culture of instructor autonomy means that we will never require instructors to teach calculus in any particular way. We need a structure that liberates instructors to chose a pedagogy that matches their strengths and helps their students achieve these goals.

We also need to articulate learning goals for each calculus course that include higher level learning in addition to specific content. These higher level learning goals could then drive what instructors are expected to cover in each calculus course, in place of the current detailed list of specific topics. The focus on high level learning goals will also provide the freedom for instructors to identify, develop, and adapt appropriate pedagogical strategies to meet those goals. To keep the sequence intact, a reduced list of topics will still be needed for the sake of subsequent courses as well as coherence within sections of the same course, but the focus will be on the higher level learning goals.

Furthermore, we need to learn about the wide variety of pedagogical approaches that may be appropriate and how to apply them to the specific context of our calculus courses. As we learn about new methods and deepen our knowledge of the possibilities, we will all benefit. Some will see new ways to incorporate conceptual ideas into their lectures, while others will be able to teach in completely new ways. By discussing these issues with colleagues at other colleges who have grappled with similar challenges, we can learn what has been successful (or not) at other institutions and adapt the ideas to our own courses here. For example, approaches that foster active learning could increase engagement of students in the mathematics itself, and writing assignments that put the calculus concepts in different applied contexts could improve both mathematical communication skills and transferability of calculus skills to other disciplines.

Once we have become informed about the pertinent pedagogical approaches and identified learning goals we all agree on, each instructor can determine teaching approaches that work best for us to meet those goals and support the full range of our students, from non-majors to math majors, in learning mathematics and mathematical ways of thinking. We will continue to highly value and support calculus instructors who have tremendous success with current teaching approaches, while opening up opportunities for instructors who want to try very different approaches.

Our efforts will include the following main tasks:

- Discuss with colleagues at other institutions what they have done to improve student success in their introductory mathematics courses, particularly calculus.
- Learn about high-impact teaching practices that have been shown to improve students' ability to apply conceptual knowledge across the boundaries between disciplines.
- Meet with other departments on campus whose courses use calculus to discuss ways to improve how well our courses serve their students.
- Articulate higher level learning outcomes for all our calculus courses that go beyond the content goals, and revise the syllabi for these courses to reflect these learning outcomes and allow for more flexibility in how instructors achieve these outcomes and support student learning.

Proposed timeline

The project period would be Fall 2018, Spring 2019, and Fall 2019. At any one time, there would be three faculty in the core project group, who may rotate on a semester basis. This group would be responsible for inviting visitors, arranging site visits, scheduling meetings, coordinating with the Center for Teaching and Learning, and writing a six-month progress report and a final report. These duties would be divided evenly among the group, so each would get the same honorarium of $1,500 per semester. Michael Ching, David Cox, and Tanya Leise will take the lead in directing the grant, with other mathematics faculty also actively involved and the entire Department taking part in discussions.

In Fall 2018, we would start inviting external visitors to campus. If the timing is right, a visitor could also give an undergraduate colloquium and interact with both students and faculty. We would also do at least one off-campus site visit in the Fall. The project group would also meet every other week to discuss teaching methods and our calculus syllabi. Some of these meetings will involve the whole Department, the Center for Teaching and Learning, or representatives of other departments on campus. The goal for this semester would be a tentative agreement on topics to remove from the calculus syllabi to allow room for deeper learning and experimentation. We would also begin the process of incorporating explicit learning goals into the syllabi.

In Spring 2019, the main goal would be to draft new versions of the syllabi for all three calculus courses. There would also be an off-campus visit focused on new pedagogies and meetings with the CTL to properly incorporate new learning goals into our syllabi. One important topic will be a serious discussion of numerical issues and technology.

In Fall 2019, we would teach MATH 211 based on the new syllabus and use feedback from students and faculty throughout the semester to make adjustments to the syllabi for all three calculus courses.

Proposed activities

As this project unfolds, instructors of the same course may begin to use different pedagogical approaches. So a student may see one approach in MATH 111 but encounter a quite different pedagogy in MATH 121. We want students to have the same foundational grasp of the material, regardless of how the course is taught. The first step in this project, therefore, is to identify the core mathematical concepts that we want all our calculus courses to cover.

We will produce a pared down syllabus for each course that highlights the most central content we want students to master, as well as higher-level habits of mind that we aim to foster in the students. By focusing on a smaller core of key ideas, we will have more time in the semester to spend on conceptual understanding and applications in other disciplines or within mathematics. Agreeing on this core will require internal discussions within our Department as well as conversations with other departments that require calculus for some of their classes.

To inform our discussions, and to learn about how other schools have addressed these issues, we will conduct external site visits to schools that have a reputation for innovative calculus teaching. Some schools we might include: Macalester College (who use ideas from multivariable calculus in courses of all levels); Westfield State University (who have a strong track record of using inquiry-based learning); Williams College (who have a calculus sequence

that handles placement in a different way).

We also plan to bring external speakers to Amherst who have done research in calculus instruction or have direct experience with the teaching practices we want to learn about. For example, visitors might include some of the authors of the CUPM Curriculum Guide or someone like Chad Topaz, who is currently at Williams but was instrumental in creating the Macalester calculus sequence.

After collecting information from other schools, and examining textbooks and other supporting material, we will begin the process of redesigning our calculus syllabi. With help of staff in the Center for Teaching and Learning, members of the Mellon group will design suitable projects and assignments that support a variety of pedagogies. We will start with MATH 211 since it is the last course in the sequence. We will then work backwards to make these new ideas available to MATH 121 and MATH 111 instructors.

In addition to considering what happens in the classroom, we will also look at the implications of these changes for academic support. This will include an examination of our Math Fellows program, peer tutors, and the roles of the Quantitative Center and Writing Center. The new teaching practices instructors may decide to use will be new for students as well. For example, many students will have no experience with larger-scale writing assignments in mathematics. Therefore, it will be particularly important to provide the appropriate support outside the classroom for students whose instructors choose to assign these activities.

A key concern as we make changes to our calculus sequence is how they will impact our placement procedures. One of the most challenging aspects of our program is the wide variety of experiences in mathematics students have had before coming to Amherst. Placing students into the appropriate class is crucial to both their success and that of the students around them. We need to plan our learning outcomes for MATH 121 and MATH 211 keeping in mind that many students come straight into those classes from high school.

Assessing the success of our project

We plan to assess the outcomes of this project through a combination of tools, with the guidance of the Center for Teaching and Learning. These tools could include the following:

- We will evaluate site visits and invited speakers for their effectiveness in conveying the desired information, and compile summary documentation of what is learned from each visit and speaker.
- Instructors who implement new pedagogical approaches will compare syllabi before and after these efforts, and map the updated learning goals onto each to assess improvements in meeting these goals.
- In-class "one-minute" reflections by students several times during the semester with questions targeting how specific pedagogical strategies are supporting their learning, as well as targeted questions on the course evaluations at the end of the semester. Qualitative analysis of the typed up transcripts of these student comments will be done with the assistance of the CTL.

Budget

Here is our proposed budget, with estimates of expected costs.

Travel expenses: $10,000

2 site visits × 3 faculty per visit × $1,000 per faculty = $6,000 for site visits
4 one-day visitors × $500 per visitor = $2,000 for one-day visitors
2 three-day visitors × $1,000 per visitor = $2,000 for three-day visitors

Honoraria for Visitors: $2,500

4 one-day visitors × $250 per day = $1,000
2 three-day visitors × $250 per day = $1,500

Honoraria for Project Group: $13,500

3 faculty members in each of three semesters ($4,500 total per semester)

Meeting Costs: $500

Total Budget: $26,500