Revisiting Multiplicity of Bubble Equilibria in a Search Model with Posted Prices*

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Abstract

Multiplicity of equilibria naturally obtains in search models of money with price posting: buyers' money holdings depend on posted prices, which, in turn, depend on buyers' money holdings. I show that this multiplicity of equilibria exists in general even when money is replaced with a dividend-bearing asset, as long as the asset is useful as a medium of exchange. If the fundamental value of the asset is sufficiently high, there is a unique equilibrium, in which the price of the asset equals its fundamental value. For lower fundamental values, there is a continuum of bubble equilibria, in which the price of the asset exceeds the fundamental value. For very low fundamental values, the equilibrium is again unique but the asset is not used as a medium of exchange. I characterize the set of symmetric and asymmetric bubble equilibria for both positive and negative fundamental values.

Keywords: Search, Money, Multiplicity, Indeterminacy, Liquidity
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1 Introduction

Assets are valued both for their fundamental value - the stream of consumption that they generate - and for their liquidity, i.e. the extent to which they are useful as media of exchange. It is now well understood that the exchange role of assets can push their price above their fundamental value. This liquidity premium on assets has been emphasized by a growing literature in monetary economics, which explicitly models the frictions that make a medium of exchange essential.[1] This literature demonstrates that the endogeneity of the asset's liquidity value often leads to multiple steady-state equilibria.

I study a particular channel through which decentralized trade can generate multiplicity of equilibria, namely a coordination problem resulting from price posting. In the model, agents use an asset to purchase an indivisible good in a frictional market. If sellers of this good post high prices, agents have an incentive to hold large amounts of the asset. This drives up the demand for the asset, which results in a high asset price. In turn, if the real value of the asset is high, sellers have an incentive to post high prices. Thus, there is a self-confirming equilibrium with a high asset price. Similarly, there is an equilibrium in which sellers post low prices, and the real value of the asset is low. Generically, whenever there exists a bubble equilibrium - one in which the asset price exceeds its fundamental value - there exists a continuum of such equilibria. I characterize the set of symmetric bubble equilibria for arbitrary fundamental values, including zero (fiat money) and negative values (e.g., a storage cost). In addition, even when a symmetric bubble equilibrium does not exist, I show that there may exist asymmetric bubble equilibria, in which some but not all agents carry the asset.

A growing literature in monetary theory, starting with Green and Zhou (1998), has established indeterminacy of steady state equilibrium in search models of fiat money. Jean *et al.* (2010) show that a continuum of steady-state monetary equilibria also exists in the price-posting version of the Lagos and Wright (2005) framework, which is more tractable than Green and Zhou (1998), and show that this indeterminacy can be interpreted as the result of a coordination problem between buyers and sellers. Since equilibrium indeterminacy seems to stem from the endogenous exchange value of money, Wallace (1998) conjectured that it disappears if fiat money is replaced by commodity money, or equivalently, by an asset that pays a dividend. I show this conjecture to be false in the Lagos and Wright (2005)

[1] Search-theoretic models in which money is essential started with Kiyotaki and Wright (1989, 1993). For recent contributions on the liquidity premium in search models, see Geromichalos *et al.* (2007), Lagos (2008, 2010; 2011), Rocheteau and Wright (2013), and the recent survey by Lagos *et al.* (2015).

framework: the indeterminacy is robust to the introduction of a positive dividend, as long as the value of the dividend is not too high. My contribution complements the work of Zhou (2003), who showed a similar robustness result in the Green and Zhou (1998) model. Furthermore, I provide a simple intuition for the multiplicity of equilibria when the dividend value is low, and the uniqueness when the dividend value is high. Specifically, when the dividend value of an asset exceeds the consumption utility of the good it is used to buy, the buyer's budget constraint does not bind at the point of sale. As a result, the demand for the asset is not determined by whether it is useful in decentralized exchange, but is instead depends only on its dividend value. This uniquely pins down the price of the asset. When the dividend value is lower than the threshold, the value of the asset in exchange - which is endogenous - matters for its price, leading to the existence of bubble equilibria, in which the asset is valued above its dividend value. For very low dividend values, the equilibrium is again unique, but in this case the asset is not used as a medium of exchange.

The indivisibility of the good traded in the decentralized market is important for the result. Intuitively, because of the indivisibility, the exchange value of the asset is discontinuous in the amount of asset holdings. I discuss the role of indivisibility in more detail in section 3.1. The nature of the price posting mechanism is also crucial. For example, there is a unique stationary equilibrium in the Lagos and Wright (2005) model if prices are instead determined by Nash bargaining, as shown in Wright (2010). In a closely related recent paper, Julien *et al.* (2014) demonstrate that, even with indivisible goods, the equilibrium is unique under either Nash bargaining or competitive search. This is because, under Nash bargaining, the terms of trade are determined after the seller has observed the buyer's asset holdings. Under competitive search, the seller commits to a price before meeting a buyer, but since buyers direct their search, the posted price affects the asset holdings choice of the buyer it attracts. In sharp contrast to Julien *et al.* (2014), a coordination problem arises in my environment because each seller posts the selling price prior to observing the asset holdings of the buyer he/she is matched with; thus, posted prices can be conditioned only on the aggregate value of asset holdings.

My result also contributes to the aforementioned literature on the liquidity role of assets as media of exchange. This literature has emphasized the self-referential nature of liquidity: for example, since the liquidity value of an asset is at least partly endogenous, expectations of high asset prices in the future can lead to high asset prices today. I identify an additional - coordination-based - channel driving equilibrium indeterminacy. As explained above, my result shows that the liquidity role of an asset is more likely to cause its price to deviate from the fundamental value - and more likely to lead to multiple equilibria - when this fundamental value is low.

Finally, my result contributes to the literature on the Diamond (1971) paradox: the result that, in search models with price posting, sellers always extract the entire surplus from trade. Green and Zhou (1998) and Jean *et al.* (2010) show that this need not hold when agents require money for trade, and their budget constraints are therefore endogenous. My result shows that the Diamond paradox reappears only if money is replaced with an asset of sufficiently high fundamental value.

2 Model

Time is discrete and the time horizon is infinite. There is a $[0,1]$ continuum of ex ante identical agents with discount factor $\beta \equiv 1/(1+r) \in (0,1)$. Each period is divided into two subperiods. In the first subperiod agents produce and consume goods in a decentralized market, DM, with random bilateral matching. In a random match between agents i and j, the probability i wants to consume the good j can produce but not vice-versa, a single-coincidence meeting, is $\sigma \in (0, \frac{1}{2}]$. The probability i wants to consume a good j can produce and vice-versa, a double-coincidence meeting, is 0. When i wants to consume what j produces, the former is called a buyer and the latter a seller. The DM good is indivisible. Let u be the utility from consuming one unit, and c the cost of producing one unit, of the indivisible good, conditional on it being one that the buyer consumes and the seller produces. I assume throughout that $0 < c < u$. In the second subperiod all agents consume a perfectly divisible good X and supply divisible labor H in a Walrasian centralized market, CM. Without loss of generality, it is assumed that H produces X one-for-one. Let $U(X) - H$ be the utility of consuming X and working H in the CM, where $U(X)$ satisfies standard assumptions.[2]

There exists an asset that can serve as a medium of exchange in the DM. The asset is storable, is available in fixed supply, A, and does not depreciate. One unit of the asset delivers δ units of X in the CM every period. Note that the fiat money case corresponds to $\delta = 0$. Denote by $\gamma = \delta/(1-\beta)$ the fundamental value of the asset. In the DM, agents post selling prices p in units of the asset.

[2]Note that I adopt the environment of Lagos and Wright (2005), in which each agent's buyer/seller status is random in each period. An alternative would to be assume, following Rocheteau and Wright (2005), that some agents are permanently sellers, while others are permanently buyers. All the results that follow hold in the Rocheteau and Wright (2005) environment as well.

3 Equilibrium

Let $V(a,p)$ be the expected payoff of an agent entering the DM with a units of the asset and a posted price of p, and $W(a)$ the expected payoff of an agent entering the CM with a. Let ϕ be the CM price of the asset in terms of X. Then

$$\begin{aligned} W(a) &= \max_{X,H,a',p'} \{U(X) - H + \beta V(a', p')\} \\ \text{s.t. } X &= H + \phi(a - a') + \delta a \end{aligned} \tag{1}$$

Substituting H from the budget constraint yields

$$W(a) = \max_{X} \{U(X) - X\} + (\phi + \delta) a + \max_{a',p'} \{-\phi a' + \beta V(a', p')\} \tag{2}$$

The optimal choice of a' is independent of a. The following envelope condition holds:

$$W'(a) = \phi + \delta \tag{3}$$

Suppose that a buyer with asset holdings a meets a seller in the DM who has posted a price $\tilde{p}$. A necessary and sufficient condition for the buyer to buy is $a \geq \tilde{p}$ (budget constraint) and $u + W(a - \tilde{p}) \geq W(a)$ (individual rationality). Equation (3) implies that trade will occur if and only if $\tilde{p} \leq R(a)$, where

$$R(a) = \min\left\{\frac{u}{\phi + \delta}, a\right\}$$

is the reservation price of a buyer with asset holdings a.

A stationary equilibrium, in general, features a distribution of posted DM prices and a distribution of asset holdings across agents in the DM, which, in turn, determines the distribution of reservation prices. Let F be the cumulative distribution of posted selling prices, and let G be the cumulative distribution of reservation prices. The payoff of an individual who chooses asset holdings a and posts price p is

$$\begin{aligned} V(a,p) &= W(a) + \sigma \int_0^{R(a)} [u + W(a - \tilde{p}) - W(a)]\, dF(\tilde{p}) \\ &\quad + \sigma(1 - G(p))(-c + W(a+p) - W(a)) \end{aligned} \tag{4}$$

Since agents always have the option of buying the asset and not using it to purchase the DM good, any equilibrium must satisfy

$$\phi \geq \beta\gamma; \tag{5}$$

the price of the asset must be at least equal to its discounted dividend value. Since I allow $\gamma < 0$, I assume free disposal of the asset, so that always $\phi \geq 0$. It is easy to see (and will be confirmed below) that an equilibrium with $\phi = \max\{0, \beta\gamma\}$ always exists. I refer to any equilibrium with $\phi > \max\{0, \beta\gamma\}$ as a *bubble* equilibrium. Most of the analysis, summarized in Results 1-3 below, focuses on symmetric bubble equilibria, in which both distributions are degenerate: all agents choose the same asset holdings, and all agents post the same selling price. Result 4 illustrates that there also exist asymmetric equilibria.

In order for the asset market to clear in a symmetric equilibrium, each agent's asset holdings must be equal to A. The two endogenous variables to be determined are then ϕ, the CM price of the asset in units of X, and p, the DM price of the indivisible good in units of the asset. For DM trade to occur - which must be the case in a bubble equilibrium - several conditions must hold. First, trade must be profitable ex post for both the buyer and the seller:

$$c \leq p(\phi + \delta) \tag{6}$$

$$p(\phi + \delta) \leq u \tag{7}$$

Second, the posted price p cannot exceed A, each agent's asset holdings. Third, since sellers pick the price to maximize profits, they post the highest price at which the buyer is willing and able to buy the indivisible good. This pins down

$$p = \min\left\{\frac{u}{\phi + \delta}, A\right\} \tag{8}$$

The last restriction on the equilibrium with DM trade is more subtle. Agents must be willing *ex ante* to carry A units of the asset into the DM. From the expressions for V and W, this is true if and only if

$$\beta\sigma\left[u - p(\phi + \delta)\right] \geq \left[\phi - \beta(\phi + \delta)\right] A \tag{9}$$

Notice that equations (5) and (9) imply (7): if buyers are willing to carry A ex ante, then they are willing to make the trade ex post.

From (5) and (8), we conclude that

$$p \leq \min\left\{\frac{u}{\gamma}, A\right\} \tag{10}$$

This motivates us to consider three regions of the parameter space; in particular, different values of γA relative to u and c result in different sets of equilibria. I consider each in turn.

The first result shows that, if the fundamental value of the asset is sufficiently large, the equilibrium is unique.

Result 1 *Suppose that $\gamma A \geq u$. Then, there is a unique symmetric equilibrium, in which DM trade occurs, $p = \frac{u}{\gamma}$, and $\phi = \beta\gamma$.*

Proof. By (5), $\frac{u}{\phi+\delta} \leq \frac{u}{\gamma}$, which, by assumption, does not exceed A. Therefore, equation (8) implies that $p = \frac{u}{\phi+\delta}$. In order for the asset market to clear, (9) needs to hold; substituting $p = \frac{u}{\phi+\delta}$ into (9) implies that $\phi + \delta \leq \gamma$, or, equivalently, $\phi \leq \beta\gamma$. Together with (5), this pins down $\phi = \beta\gamma$. ■

If the fundamental value of the asset is large enough, then the buyer's budget constraint cannot bind in the DM. Knowing this, the sellers charge the highest price that buyers are willing to pay ex post. This case is analogous to the Diamond (1971) result that the seller extracts the entire ex post surplus from trade. Moreover, because the buyer's surplus from DM trades is zero, demand for the asset does not depend on its usefulness in exchange in the DM, and therefore the price of the asset, ϕ, is uniquely pinned down by its fundamental value. In particular, the equilibrium price of the asset does not depend on σ.

I next consider the intermediate case, in which the fundamental value of the asset is enough to compensate the seller for the production cost in the DM, but smaller than the buyer's utility of consumption in the DM.

Result 2 *Suppose that $c \leq \gamma A < u$. Then, there is a continuum of symmetric equilibria. In every symmetric equilibrium, $p = A$, and ϕ is consistent with equilibrium if and only if*

$$\phi \in \left[\beta\gamma, \frac{\sigma u + (1-\sigma)\,\delta A}{(r+\sigma)\,A}\right]$$

Proof. Condition (5) implies $\phi + \delta \geq \gamma$. Since $\gamma A < u$, condition (8) implies $p = A$. Next, after substituting $p = A$, (9) is simplified to yield

$$\phi \leq \frac{\sigma u + (1-\sigma)\,\delta A}{(r+\sigma)\,A} \tag{11}$$

This gives an upper bound on ϕ. The lower bound is, as usual, given by (5). It is easy to check that, if $\gamma A < u$, the upper bound is greater than or equal to $\beta\gamma$, with equality if $\sigma = 0$. Finally, since $c \leq \gamma A$, any ϕ in the indicated range satisfies $c \leq A\,(\phi + \delta)$. ■

In this case, there is a continuum of equilibria, the lowest one of which has the asset priced at fundamental value. All other equilibria are bubble equilibria, in which sellers charge a price of the indivisible good that is - in terms of CM consumption - higher than the fundamental value of the asset. The buyer's budget constraint binds in the DM, and demand for the asset on the margin is determined by the price that the seller charges, not the asset's fundamental value. There thus exist multiple equilibria, despite the fact that the fundamental value of the asset is positive. Note that all the equilibria in Result 2 - and in Result 3 below - feature $p = A$. At first glance, this seems to contradict the intuition given in the introduction, which is that the price of the DM good, and hence demand for the asset, varies across equilibria. However, the price of the DM good in units of CM consumption, ϕA, does vary across equilibria. Thus, we could equivalently consider an environment (as done in Rocheteau and Wright (2005) and Jean *et al.* (2010)) in which sellers post prices in "real" terms - i.e., in units of CM consumption, rather than in units of the asset. In such an environment, a higher real posted price of the DM good justifies higher real asset holdings, and vice versa, leading to exactly the same multiplicity of equilibria.

Finally, I consider the case in which the dividend value of the asset is not enough to pay the cost of producing the DM good.

Result 3 *Suppose that $\gamma A < c$. Then there exists a symmetric equilibrium in which $\phi = \max\{0, \beta\gamma\}$ and DM trade does not occur. In addition, define*

$$\omega \equiv \frac{(r+\sigma)\,c - \sigma u}{r}$$

1. *If $\gamma A < \omega$, there are no other symmetric equilibria.*
2. *If $\gamma A \geq \omega$, there is a continuum of symmetric bubble equilibria in which DM trade occurs with $p = A$ and*
$$\phi \in \left[\frac{c - \delta A}{A}, \frac{\sigma u + (1-\sigma)\,\delta A}{(r+\sigma)\,A}\right]$$
Note that the lower bound of this interval is strictly above $\beta\gamma$.

Proof. First, we show the existence of an equilibrium with $\phi = \max\{0, \beta\gamma\}$. Suppose $\gamma > 0$; if there is no DM trade and $\phi = \beta\gamma$, agents are indifferent over holding any quantity of the asset, and the asset market therefore clears in the CM. If $\gamma < 0$, then $\phi = 0$. If $\phi = \max\{0, \beta\gamma\}$ and $\gamma A < c$, there does not exist a feasible price at which sellers wish to sell, so the allocation without DM trade is in fact an equilibrium. Next, consider equilibria with DM trade. In any such equilibrium, (5), (8) and $\gamma A < c < u$ imply that $p = A$. In order for sellers to be willing to sell, we must have $c - \delta A \leq \phi A$. This gives a lower bound on ϕ. The upper bound on ϕ is (11). In order for such an equilibrium to exist, we must therefore have

$$c - \delta A \leq \frac{\sigma u + (1-\sigma)\,\delta A}{(r+\sigma)},$$

which can be rewritten as $\gamma A \geq \omega$. ■

The quantity ω defined above determines the smallest fundamental value of the asset for which it can serve as a medium of exchange. Notice that $\omega < c$, and so the condition $\gamma A < \omega$ is stronger than $\gamma A < c$. Intuitively, if $\gamma A < \omega$, the asset is insufficient to compensate for the seller's cost of producing, even if it can be used to buy consumption in the future.

There are three important corollaries to Result 3. First, the fiat money case falls into this category. When $\delta = 0$, there exists an equilibrium in which money is not valued ($\phi = 0$) and DM trade does not occur. If $c \leq \frac{\sigma u}{r+\sigma}$ (or, equivalently, $\omega \leq 0$), Jean *et al.* (2010) show that there also exist a continuum of equilibria with valued money. The present result is a strict generalization of theirs. Jean *et al.* (2010) also discuss ways to break the multiplicity result by changing the timing of the model. In particular, they show that the equilibrium is unique in a modified setting where the number of sellers is finite and the game between buyers and sellers is sequential, rather than simultaneous. This result carries over to my setting in a straightforward way, and hence I omit a formal discussion of it.

Second, Result 3 accommodates the possibility that $\delta < 0$. This is relevant if, for example, the asset has a positive storage cost. In particular, if $\omega \leq \gamma A \leq 0$, there exists a continuum of equilibria that with $\phi > 0$, despite the fact that the fundamental value of the asset is negative.

Third, when there is a continuum of equilibria, the lower bound on the range of equilibria is $\phi = \frac{c-\delta A}{A}$, which gives ϕ as a *decreasing* function of δ. To understand the negative dependence of ϕ on δ, recall that the lower bound on the range of ϕ is determined by the seller's indifference condition $c = (\phi + \delta)\,A$. A seller is willing to hold the asset either for its dividend value or for its (ex-dividend) price. So, if δ increases, the equilibrium ϕ must decrease one-for-one to keep the seller indifferent to accepting the asset. The result that higher fundamental value is compensated with a lower liquidity value is common in search models, going back to Kiyotaki and Wright (1993). In recent work, Julien *et al.* (2014) find a similar negative relationship between δ and ϕ under alternative pricing mechanisms, such as Nash bargaining and competitive search. Note that the same logic does not hold for the upper bound on ϕ, which is increasing in δ. The asymmetry is due to the fact that, while the lower bound is determined by an *ex post* indifference condition, the upper bound is determined by the buyer's *ex ante* indifference condition. An increase in δ makes agents more willing to hold the asset *ex ante*, thus raising the maximum price they are willing to pay for it.

In summary, I have established that a positive fundamental value does not eliminate the multiplicity of bubble equilibria unless this fundamental value is large enough - more precisely, so large that the buyer's budget constraint is not binding in the DM. For an intermediate range of fundamental values, the set of symmetric bubble equilibria is an interval, with the fundamental value being its lower bound. Finally, for fundamental values below the cost of the DM good, the range of ϕ for which DM trade occurs is an interval that does not include $\phi = \beta\gamma$. These results are illustrated graphically in Figure 1 for the case where $\omega < 0$. The region labeled S shows the set of symmetric bubble equilibria, displaying the range of possible equilibrium values of ϕ for each γ. It is bounded below by the fundamental value $\phi = \beta\gamma$, above by the buyer's indifference condition (11), and on the left by the seller's indifference condition $\phi = \frac{c-\delta A}{A}$. Figure 2 similarly illustrates the set of bubble equilibria for the case $\omega > 0$. Note that, in this case, there is no bubble equilibrium when the asset is fiat money, i.e. no monetary equilibrium.

The above analysis has demonstrated the multiplicity of symmetric bubble equilibria, in which the distribution of asset holdings is degenerate at A. However, the environment also allows for many other equilibria. To illustrate this, I show how to construct asymmetric bubble equilibria, in which not all agents carry the asset. Since I focus on bubble equilibria, I assume that $\gamma A < u$. Specifically, I consider equilibria in which: $\phi > \beta\gamma$; a fraction $n \in (0,1)$ of the agents each carry A/n units of the asset, whereas the remaining $1-n$ each carry zero; and the price of the DM good is $p = A/n$. Thus, I look at equilibria in which the choices of asset holdings are asymmetric, but the distribution of prices is still degenerate. The existence of equilibria with a non-degenerate distribution of prices is an open question and is left for future work.

The key observation is that, in any such equilibrium, agents must be indifferent between carrying p units of the asset and carrying zero. This pins down $\beta\sigma\left[u - p\left(\phi+\delta\right)\right] = \left[\phi - \beta\left(\phi+\delta\right)\right]p$, which, after substituting $p = A/n$, simplifies to

$$\phi = \frac{\sigma u + (1-\sigma)\,\delta\left(\frac{A}{n}\right)}{(r+\sigma)\left(\frac{A}{n}\right)} \tag{12}$$

It is easy to verify from (12) that $n \leq 1$ and $\phi > \beta\gamma$ are only possible if $\gamma A < u$. Next, since any equilibrium must satisfy $c \leq (\phi+\delta)\,p \leq u$, this implies, after re-arranging,

$$(r+\sigma)\,c - \sigma u \leq (1+r)\,\delta\left(\frac{A}{n}\right) \leq ru \tag{13}$$

This provides upper and lower bounds on n, which, in turn, uniquely determines $p = A/n$ and then ϕ via equation (12). This establishes the following result:

Result 4 *Suppose that $\gamma A < u$. Then, there exist asymmetric bubble equilibria, in which: a fraction n of the agents hold A/n units of the asset, while the remaining $1-n$ hold zero; the price of the DM good is $p = A/n$; and ϕ is given by equation (12). Specifically:*

1. *If $\delta > 0$ and $\omega \leq \gamma A < u$, then n is consistent with equilibrium if and only if*
$$\frac{\gamma A}{u} \leq n \leq 1,$$
and the corresponding ϕ satisfies $\beta\gamma \leq \phi \leq \frac{\sigma u+(1-\sigma)\delta A}{(r+\sigma)A}$.

2. *If $\delta > 0$ and $0 < \gamma A < \omega$, then n is consistent with equilibrium if and only if*
$$\frac{\gamma A}{u} \leq n \leq \frac{\gamma A}{\omega},$$
and the corresponding ϕ satisfies $\beta\gamma \leq \phi \leq \frac{\sigma u+(1-\sigma)c}{(1+r)\omega}\gamma$.

3. *If $\delta = 0$ and $(r+\sigma)\,c \leq \sigma u$, then any $n \in (0,1]$ is consistent with equilibrium, with $0 \leq \phi \leq \frac{\sigma u}{(r+\sigma)A}$. If $\delta = 0$ and $(r+\sigma)\,c > \sigma u$, then no n is consistent with equilibrium.*

4. *If $\delta < 0$ and $\omega \leq \gamma A$, then n is consistent with equilibrium if and only if*
$$\frac{\gamma A}{\omega} \leq n \leq 1,$$
and the corresponding ϕ satisfies $\frac{\sigma u+(1-\sigma)c}{(1+r)\omega}\gamma \leq \phi \leq \frac{\sigma u+(1-\sigma)\delta A}{(r+\sigma)A}$.
If $\delta < 0$ and $\omega > \gamma A$, then no n is consistent with equilibrium.

Proof. This follows directly from combining (13) with the restriction $0 < n \leq 1$. ■

Intuitively, a lower bound on p is equivalent to an upper bound on n, and vice versa. Suppose that $\delta \geq 0$. Since $p = A/n$, n needs to be sufficiently high (i.e. p needs to be sufficiently low) for the buyers to be willing to buy the DM good; and n needs to be sufficiently low (i.e. p needs to be sufficiently high) for the sellers to be willing to sell. On the other hand, when $\delta < 0$, there is a cost to carrying the asset, and so the sellers' indifference condition now determines the lower bound on n (if n is too low, sellers obtain too much of the asset).

Figures 1 and 2 illustrate the set of equilibria for $\omega < 0$ and $\omega > 0$, respectively. In the region S, γ is such that both symmetric and asymmetric equilibria exist. The area labeled

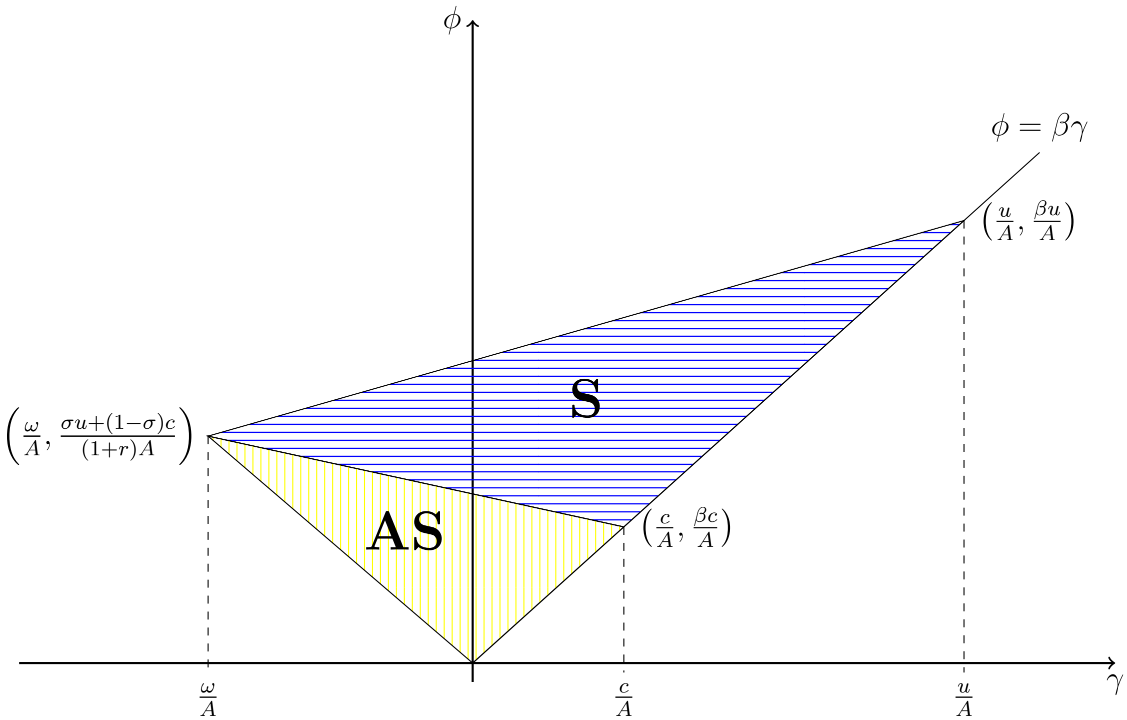

Figure 1: The $\omega < 0$ case. S: symmetric and asymmetric equilibria exist. AS: only asymmetric equilibria exist.

AS, on the other hand, shows the region in which a symmetric bubble equilibrium does not exist, but an asymmetric one still does. When $\omega < 0$, Figure 1 illustrates that allowing for asymmetric equilibria increases the range of equilibrium ϕ (items 3 and 4 of Result 4). Even more interestingly, when $\omega > 0$, asymmetric bubble equilibria exist for a wider range of parameters than symmetric bubble equilibria, as illustrated in Figure 2 and item 2 of Result 4. When $0 < \gamma A < \omega$, Result 3 implies that there does not exist an equilibrium with $n = 1$ (i.e. a symmetric one), but there still exist equilibria with $n < 1$, because lower values of n imply higher values of p. In other words, a lower n increases the quantity of the asset handed over to the seller in each transaction, thus making the seller more willing to accept the trade. Thus, for any positive δ, there exists an n consistent with equilibrium, since a sufficiently low n implies a sufficiently high p for the sellers to be willing to sell.

3.1 Discussion: the role of indivisibility

This paper has established that the endogeneity of buyers' budget constraints overturns the result of Diamond (1971) that sellers always extract the full surplus from buyers, and leads to the possibility of multiple equilibria, even with intrinsically valued assets.

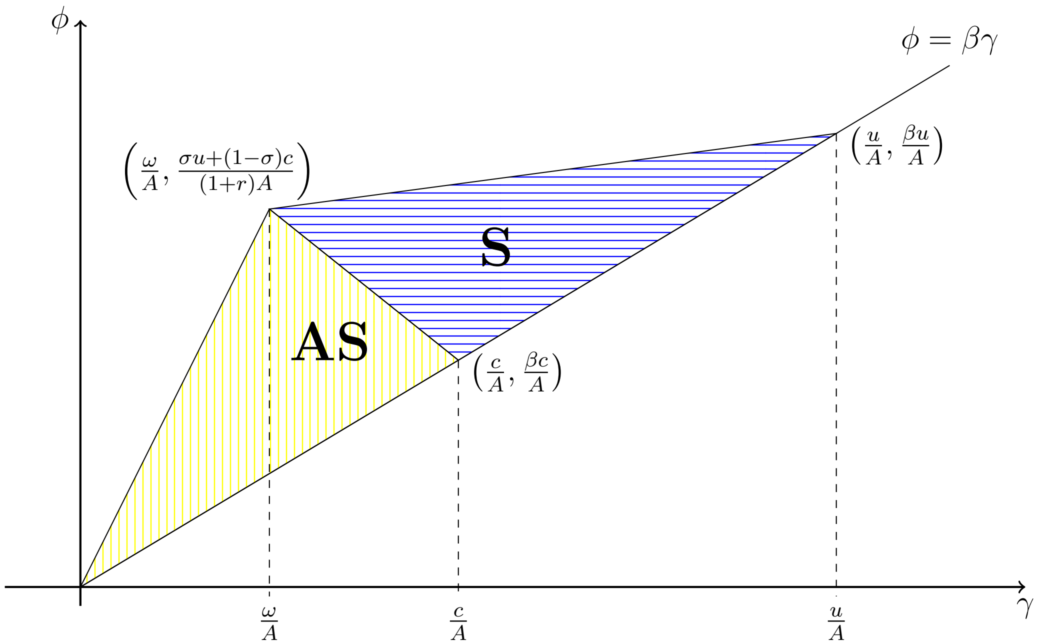

Figure 2: The $\omega > 0$ case. S: symmetric and asymmetric equilibria exist. AS: only asymmetric equilibria exist.

As mentioned in the introduction, the indivisibility of the DM good is important for these results. To illustrate this, I now consider a modification of the model in which this good is divisible: the seller's cost of producing a quantity q is an increasing and convex function $c(q)$, and the buyer's utility of consuming a quantity q is an increasing and concave function $u(q)$. Assuming that both functions are twice differentiable, define the ex post efficient quantity q^* to be the unique solution to $u'(q^*) = c'(q^*)$.

Instead of posting a price, sellers now post pairs (q, p), where q is the quantity of the DM good produced, and p is the amount of the asset transferred from the buyer to the seller. This is without loss of generality: sellers could post menus of such pairs, but we only need to focus on the pair optimally selected by the buyer; this pair is unique, since all the buyers have identical preferences. Knowing that all the buyers hold A units of the asset, the seller would solve

$$\max_{q,p} p(\phi + \delta) - c(q)$$

subject to the buyer's individual rationality constraint

$$u(q) - p(\phi + \delta) \geq 0 \tag{14}$$

and the buyer's budget constraint

$$p \leq A \tag{15}$$

It is easy to see that (14) always binds. If it holds with strict inequality, the seller could increase profits by slightly lowering q, without violating the constraint. Therefore, the seller sets $q = \min\left\{q^*, u^{-1}\left(A\left(\phi+\delta\right)\right)\right\}$ and $p = \frac{u(q)}{\phi+\delta} = \min\left\{\frac{u(q^*)}{\phi+\delta}, A\right\}$. If $\gamma A \geq u\left(q^*\right)$, we have $\phi = \beta\gamma$, $q = q^*$, and $p = \frac{u(q^*)}{\gamma}$. If $\gamma A < u\left(q^*\right)$, we have $p = A$ and $u\left(q\right) = A\left(\phi+\delta\right)$, leaving the buyer with zero surplus ex post. It immediately follows that, if $\gamma A < u\left(q^*\right)$, the asset must be valued at the fundamental value, since its use in exchange creates no additional surplus for the buyer. Therefore, the equilibrium is still unique and has $\phi = \beta\gamma$, $p = A$, and $u\left(q\right) = \gamma A$.

In the above argument, the price of the asset always equals its fundamental value because DM trade generates zero surplus for the buyer. A growing literature suggests that this zero-surplus result is fragile, even with divisible goods. For example, Head *et al.* (2012) and Bethune *et al.* (2016) allow buyers to observe multiple prices or occasionally contact multiple sellers in the DM, as in Burdett and Judd (1983) and Lester (2011). Ennis (2008) and Dong and Jiang (2014) demonstrate that the zero-surplus result disappears when buyers have private information about their preferences. Revisiting the multiplicity of equilibria in these environments with intrinsically valued assets is a potential avenue for future research.

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