1. (9 points) Evaluate each of the following integrals:

(a) \( \int \frac{\sin(2x)}{\cos^2(2x)} \, dx \)  
(b) \( \int x^3 \ln(x^3) \, dx \)  
(c) \( \int_1^2 \frac{d}{dx} e^{x^4} \, dx \)

2. (8 points) For each of the following improper integrals, determine whether or not it converges. In any case of convergence, evaluate the integral.

(a) \( \int_0^3 \frac{1}{\sqrt{9-x^2}} \, dx \)  
(b) \( \int_3^\infty \frac{1}{x^2 - 3x + 2} \, dx \)

3. (8 points) For each case below, determine whether or not the limit exists. If it does, find its value.

(a) \( \lim_{x \to 0} \frac{1}{x^2} - \frac{\sinh x}{x^3} \)  
(b) \( \lim_{x \to \infty} \frac{1}{(1+x^2)\ln x} \)

4. (8 points) Let \( R \) be the region beneath the graph of \( y = \cos x \) and above the x-axis, for \( 0 \leq x \leq \frac{\pi}{2} \).

(a) If \( R \) is rotated about the y-axis, determine the volume of the resulting solid.

(b) If \( R \) is rotated about the line \( y = -1 \), determine the volume of the resulting solid.

5. (9 points) Consider the curve defined by \( x = \tan t - t, \quad y = \ln(\cos t), \quad 0 \leq t \leq \frac{\pi}{3} \).

(a) Determine the slope of this curve when \( t = \frac{\pi}{4} \).

(b) In which quadrant does this curve lie (ignore its endpoints).

(c) Find the length of this curve.

6. (6 points) The curve \( C \) is, by \( y = 2\sqrt{x}, \quad 0 \leq x \leq 3 \), is revolved about the x-axis.

Find the area of the resulting surface.
7. (12 points) Consider the two curves defined by \( r = \sin 2\theta \) and \( r = \cos \theta \).
   
   (a) Sketch these two curves, and label with polar coordinates their points of intersection.
   
   (b) Find the area of the region that is inside both of these curves.

8. (6 points) Consider the series \( \sum_{n=1}^{\infty} (-1)^n \frac{p^n}{n^p} \).
   
   For which values of \( p \) does this series converge absolutely? Converge conditionally? Diverge?
   
   Be sure to give reasons for your assertions.

9. (9 points) In each case determine whether the given series converges absolutely, converges conditionally or diverges. GIVE REASONS.
   
   (a) \( \sum_{n=1}^{\infty} (-1)^n \sin \left( \frac{1}{n} \right) \)
   
   (b) \( \sum_{n=1}^{\infty} \frac{1+2^n + 3^n + 4^n}{5^n} \)
   
   (c) \( \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{3^n (n!)^2} \)

10. (7 points) Find all values of \( x \) for which the series \( \sum_{n=1}^{\infty} \frac{\sqrt{n}(x+1)^n}{2^n (n+3)} \) converges. GIVE REASONS.

11. (8 points) By consideration of the Maclaurin series for the function \( e^{-x^2} \), determine the values at \( x = 0 \) of the fourth and of the fifth derivative of this function.

12. (10 points) (On the geometric series)
   
   (a) Find the first four nonzero terms of the series in powers of \( x \) for the function \( f(x) = \frac{1}{(2+x)^2} \).
   
   (b) Use power series to give an estimate of \( \int_{0}^{1/2} \frac{x^2}{1+x^4} \, dx \), with error < .001.