1. The function $f$ has domain $[-1,0]$ and is given by $f(x) = x^2 + 1$.

   (a) (2 points) Sketch a graph of the function $f$, labelling your axes completely.

   (b) (2 points) Use your answer to part (a) to sketch a graph of the inverse function $f^{-1}$, again labelling your axes completely. Explain how you obtained your answer.

   (c) (2 points) Find a formula for the inverse function $f^{-1}$. Explain how you got your answer.

   (d) (2 points) State the domain and range of $f^{-1}$.

   (e) (2 points) The function $g$ satisfies $g(1) = 2$, $g(2) = 5$, $g'(1) = -7$ and $g'(2) = 3$. Calculate $(g^{-1})'(2)$.

2. (6 points each) Evaluate each of the following integrals (or show that it diverges):

   (a) $\int_0^1 \frac{e^x}{\sqrt{e^x - 1}} \, dx$

   (b) $\int_2^\infty \frac{3x}{x^2 + x - 2} \, dx$

3. (3 points each) Evaluate each of the following limits:

   (a) $\lim_{x \to 0} \frac{\sin^{-1}(x)}{\sinh^{-1}(x)}$

   (b) $\lim_{n \to \infty} \frac{e^{n+1}}{n!}$

   (c) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{\sqrt{x}}$

4. (4 points each) Decide if each of the following series converges absolutely, converges conditionally, or diverges. Justify your answers.

   (a) $\sum_{n=1}^\infty \frac{(-1)^n \sin(n + 1)}{n^{3/2} + 1}$

   (b) $\sum_{n=1}^\infty \frac{(2n)!}{n!n!}$

   (c) $\sum_{n=3}^\infty \frac{(-1)^n \ln n}{n}$
5. (9 points) Find the interval and radius of convergence of the power series
\[ \sum_{n=0}^{\infty} \frac{3^{n+1}(x-2)^n}{n+2}. \]

6. (a) (8 points) Find a power series centered at \( x = 0 \) that converges to \( \tanh^{-1}(x) \).

State the range of values of \( x \) for which your answer is valid. (Hint: what is the derivative of \( \tanh^{-1}(x) \)?)

(b) (2 points) Use the **first two nonzero** terms of your answer to part (a) to estimate the value of \( \tanh^{-1}(0.03) \).

7. (a) (6 points) Find the first four terms (up to and including the term involving \( x^3 \)) in the Maclaurin series for \( f(x) = \tan x \).

(b) (2 points) Use your answer to part (a) to calculate the limit
\[ \lim_{x \to 0} \frac{\tan x - x}{x^3}. \]

8. (a) (3 points) Sketch the curve given by the parametric equations
\[ x = 1 - t, \quad y = t^3 \quad \text{for} \quad 0 \leq t \leq 2. \]

(b) (7 points) Find the area of the surface obtained by rotating the curve from part (a) around the \( x \)-axis. (You do not need to simplify your answer.)

9. (a) (5 points) Sketch the curve given in polar coordinates by the equation
\[ r = \sin \theta \quad \text{for} \quad 0 \leq \theta \leq \frac{\pi}{2}. \]

(b) (5 points) Find the area enclosed between the curve in part (a) and the \( y \)-axis.

10. (10 points) Use the Integral Test to find an upper bound for the value of the series
\[ \sum_{n=1}^{\infty} \frac{1}{n^2 + 4n + 5}. \]