• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need not simplify algebraically complicated answers. However, numerical answers such as \( \sin \left( \frac{\pi}{6} \right) \), \( 4^{\ln 4} \), \( e^{\ln 5} \), \( e^{3\ln 3} \), \( \arctan(\sqrt{3}) \), or \( \cosh(\ln 3) \) should be simplified.

• Please show all of your work and justify all of your answers. (You may use the backs of pages for additional work space.)

1. [15 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, \( +\infty \) or \( -\infty \), or Does Not Exist.

   (a) \( \lim_{x \to 0} \frac{\cosh(4x) - 1 - \arctan(4x) + 4x}{\ln(1 - x) + \arcsin x} \)

2. [20 Points] Evaluate each of the following integrals.

   (a) \( \int \frac{x^4 + 5x^2 - x + 3}{x^3 + 3x} \, dx \)

   (b) \( \int_{\frac{3\sqrt{3}}{2}}^{\infty} \frac{1}{\sqrt{36 - x^2}} \, dx \)

   (c) \( \int_{-1}^{0} x^3 \sqrt{1 - x^2} \, dx \) using a trigonometric substitution

3. [30 Points] For each of the following improper integrals, determine whether it converges or diverges. If it converges, find its value.

   (a) \( \int_{7}^{\infty} \frac{1}{x^2 - 8x + 19} \, dx \)

   (b) \( \int_{0}^{e^5} \frac{1}{x[25 + (\ln x)^2]} \, dx \)

   (c) \( \int_{-\infty}^{\infty} \cosh x \, dx \)

   (d) \( \int_{0}^{1} \ln x \, dx \)

4. [15 Points] Find the sum of each of the following series (which do converge):

   (a) \( \sum_{n=1}^{\infty} \frac{(-1)^n 3^{n+2}}{2^{4n-1}} \)

   (b) \( \sum_{n=0}^{\infty} \frac{(-1)^n (\ln 8)^n}{3^{n+1} n!} \)

   (c) \( \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{9^n (2n+1)!} \)

   (d) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots \)

   (e) \( \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n)!} \)

5. [35 Points] In each case determine whether the given series is absolutely convergent, conditionally convergent, or divergent. Justify your answers.

   (a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n + \sqrt{n}} \)

   (b) \( \sum_{n=1}^{\infty} \frac{\ln n}{n^2} \)

   (c) \( \sum_{n=1}^{\infty} \frac{n + 3}{\ln(n + 3)} \)

   (d) \( \sum_{n=1}^{\infty} \frac{\arctan(7n)}{7^n} + \frac{7}{n^7 + 1} \)

   (e) \( \sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^7 + 5} \)

   (f) \( \sum_{n=1}^{\infty} \frac{(-1)^n \pi^n}{n^n (4^n) n!} \)
6. [15 Points] Find the **Interval** and **Radius of Convergence** for the following power series
\[ \sum_{n=0}^{\infty} \frac{(-1)^n (3x + 2)^n}{(n + 1) 4^n} \]. Analyze carefully and with full justification.

7. [10 Points]  
(a) Write the first 6 non-zero terms of the MacLaurin Series for 
\[ f(x) = \sin(x^3) + \cos(x^3) \].
(b) Use this series to determine the sixth, seventh, eighth and ninth derivatives of 
\[ f(x) = \sin(x^3) + \cos(x^3) \] evaluated at \( x = 0 \).
(Hint: Do not compute out those derivatives manually.)
(Hint: Write out the definition of the MacLaurin Series for any \( f(x) \).)

8. [15 Points] Please analyze with detail and justify carefully.
(a) Write the MacLaurin series representation for \( f(x) = x \arctan(x^2) \). Your answer should be in sigma notation \( \sum_{n=0}^{\infty} \).
(b) Use the MacLaurin series representation for \( f(x) = x \arctan(x^2) \) from Part(a) to 
\[ \text{Estimate} \quad \int_{0}^{1} x \arctan(x^2) \, dx \text{ with error less than } \frac{1}{50} \]. Justify in words that your error is indeed less than \( \frac{1}{50} \).

9. [15 Points]
(a) Consider the region bounded by \( y = e^x + 2, \ y = \sin x, \ x = 0 \) and \( x = \pi \). Rotate the region about the vertical line \( x = -2 \). Set-Up but DO NOT EVALUATE the integral representing the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.
(b) Consider the region bounded by \( y = \ln x, \ y = 1, \ x = 4 \). Rotate the region about the vertical line \( x = 5 \). Set-Up but DO NOT EVALUATE the integral representing the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.
(c) Consider the region bounded by \( y = \arctan x, \ y = 0, \ x = 0 \) and \( x = 1 \). Rotate the region about the \( y \)-axis. COMPUTE the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

10. [15 Points] Consider the Parametric Curve represented by \( x = t - e^t \) and \( y = 1 - 4e^{\frac{t}{2}} \).
(a) COMPUTE the **arclength** of this parametric curve for \( 0 \leq t \leq \ln 5 \).
(b) Set-Up but DO NOT EVALUATE the **surface area** obtained by rotating this same curve about the \( y \)-axis, for \( 0 \leq t \leq 1 \).

11. [15 Points] Compute the **area** bounded outside the polar curve \( r = 2 + 2 \sin \theta \) and inside the polar curve \( r = 6 \sin \theta \). Sketch the Polar curves.