NAME:________________________________________

Read This First!

- Please read each question carefully. Show ALL work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.

- Calculators are not allowed and you must show all your work in order to receive credit on the problem.

Grading - For Administrative Use Only

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1. Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

   (a) $\lim_{x \to 0} \frac{1}{e^x - 1} - \frac{1}{x}$

   (b) $\lim_{x \to \infty} x(2\arctan(3x) - \pi)$
2. Evaluate each of the following integrals. \[ 20 \]

(a) \[ \int \frac{2x^3 + 3x^2 + 3}{2x^3 + 2x} \, dx \]

(b) \[ \int x^3 \sqrt{1 - x^2} \, dx \]
(c) \( \int_{e^{\sqrt{3}}}^{e^3} \frac{1}{x(9 + (\ln(x))^2)} \) 

(d) \( \int_{0}^{33} \frac{1}{(1 - x)^{\frac{3}{2}}} \) 

NOTE: This is an improper integral.
3. Find the sum of each of the following series. You do not need to show that they converge.

(a) \[ \sum_{n=0}^{\infty} \frac{\pi^n}{n!} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^{3n}}{2^{3n+1}} \frac{3^{n-1}}{2^{3n+1}} \]

(c) \[ -\frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - \frac{\pi^{10}}{10!} + \ldots \]
4. In each case determine whether the given series is absolutely convergent, conditionally convergent, or diverges. Justify your answers.

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^n \pi^n 4^{3n} n!}{(3n)!} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n}{n + \sqrt{n} + \sqrt[3]{n}} \]
(c) \[ \sum_{n=0}^{\infty} \frac{\sin(n)}{2n^9 - n^6 + n^3} \]

(d) \[ \sum_{n=1}^{\infty} (-1)^n e^{\frac{1}{n}} \]
5. Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

\[
\sum_{n=1}^{\infty} (-1)^n \frac{(2x - 1)^n}{3^n \sqrt[n]{n}}
\]
6.  (a) Write the first six non-zero terms of the MacLaurin Series for \( f(x) = \sin(x^3) + \cos(x^3) \).

(b) Use this series to determine the sixth, seventh, eighth and ninth derivatives of \( f(x) = \sin(x^3) + \cos(x^3) \) at \( x = 0 \).

(Hint: Do not compute out those derivatives manually.)
(Hint: Write out the definition of the MacLaurin Series for any \( f(x) \).)
7.  (a) Write the MacLaurin series representation for $f(x) = \sin(x^2)$. Your answer should be in sigma notation. 

(b) Use the MacLaurin series representation from part (a) to estimate 

$$\int_{0}^{1} \sin(x^2) \, dx$$

with error less than 0.001.
8. (a) Let R be the region bounded by the curves \( y = e^{2x}, x = \ln(3), x = 0, \) and \( y = 0. \) Rotate the region about the y-axis. **COMPUTE** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

(b) Let R be the region bounded by the curves \( y = e^{2x}, x = \ln(3), \) and \( y = 1. \) Rotate the region about the line \( y = -1. \) **Set-Up** but **DO NOT EVALUATE** the volume of this solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.
9. Consider the parametric curve given by \( x = \sin^3(t) \) and \( y = \cos^3(t) \) from \( t = 0 \) to \( t = \frac{\pi}{2} \). [20]

(a) Find the tangent line to the curve at \( \left( \frac{3\sqrt{3}}{8}, \frac{1}{8} \right) \).

(b) Find the length curve.
10. Compute the area bounded outside the polar curve \( r = 2 + 2 \cos(\theta) \) and inside the polar curve \( r = 6 \cos(\theta) \). Sketch the curves.