MATH 211 Spring 2014 Final Exam

NAME:

Show your work, and justify all your answers to avoid partial or complete loss of points. Partial credit will be given for solutions on the right track that might not be complete. Therefore it is recommended that you attempt all problems.

The problems are organized by section, not by order of difficulty.

1. Find the length of the curve $\mathbf{r}(t) = \langle \frac{e^t+e^{-t}}{2},\ \frac{e^t-e^{-t}}{2},\ t\rangle$ for $t \in [0,\ \ln 2]$. (Simplify fully.)
2. A particle moves along the curve $y = x^2 + x^3$. Find the curvature and unit tangent vector along that curve at $(1,2)$. If the acceleration of the particle at the point $(1,2)$ is $\mathbf{a} = \langle 3,\ -1\rangle$, find its tangential and normal accelerations. What is the speed of the particle at that point?
3. Let f be the following function,
$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}},\ (x,y) \neq (0,0) \\ 0,\ (x,y) = (0,0) \end{cases}$$
Show that f is continuous at $(0,0)$ but is not differentiable at $(0,0)$.
4. Let $z(x,y)$ be a solution to the equation $z^5 + xz - y = 0$ such that $z(1,-2) = -1$. Find the linearization of $z(x,y)$ near $(1,-2)$ and use it to estimate $z(0.9,-1.98)$. (Simplify fully.)
5. Find the extreme values of the function $f(x,y) = yx^2$ on the region
$$D = \{(x,y) | x \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 4\}$$
6. Find the extreme values of $f(x,y,z) = x - 2y + 2z$ on the sphere $x^2 + y^2 + z^2 = 1$.
7. Set up the following multiple integral in spherical coordinates but do not evaluate it,
$$\int\int\int_E xyz dV$$
where E is the solid region bounded by the cone $z = \sqrt{3}\sqrt{x^2+y^2}$ and the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ above the xy-plane.
8. Find the area of the portion of the circular cylinder $x^2+y^2 = 2$ below the elliptic paraboloid $z = x^2 + 2y^2$ and above the x,y-plane. Set up but do not evaluate the integral for the area of the portion of the paraboloid that lies inside the cylinder.
9. Let $\mathbf{F}(x,y) = (4x^3y^2 - 2xy^3 + x)\mathbf{i} + (2x^4y - 3x^2y^2 + 8y^3)\mathbf{j}$. Show that the vector field $\mathbf{F}$ is conservative, and use this fact to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve $C : \mathbf{r}(t) = (t + \sin(2\pi t))\mathbf{i} + (2t - \cos(2\pi t))\mathbf{j},\ 0 \leq t \leq 1$.
10. Use Green's theorem to evaluate $\int_{\mathcal{C}} (x+y)^2 dx - (x^2+y^2) dy$ where $\mathcal{C}$ is the negatively oriented triangle with the vertices $(0,0)$, $(1,1)$, and $(0,1)$.
11. **Bonus:** Reparametrize the curve of question 1 with respect to arclength measured from $t = 0$.
12. **Bonus:** Find an equation of the plane that passes through the point $(3,2,1)$ and cuts off the smallest volume in the first octant.