This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

You need not simplify algebraically complicated answers. However, numerical answers such as \( \sin \left( \frac{\pi}{6} \right) \), \( 4^\ln 4 \), \( \ln(e^7) \), \( e^{-\ln 5} \), \( e^{3\ln 3} \), \( \arctan(\sqrt{3}) \), or \( \cosh(\ln 3) \) should be simplified.

Please show all of your work and justify all of your answers. (You may use the backs of pages for additional work space.)

1. [15 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, \(+\infty\) or \(-\infty\), or Does Not Exist.

(a) \( \lim_{x \to 0} \frac{\ln(1-x) + x}{\cosh(4x) - \arctan(3x) - e^{-3x}} \)

(b) \( \lim_{x \to \infty} \left( e^{\frac{1}{3x}} - \frac{5}{x^3} \right)^{x^3} \)

2. [30 Points] Evaluate each of the following integrals.

(a) \( \int \frac{x^4 + 3x^2 + 6x^2 + 6x + 5}{x^3 + x^2 + 2x + 2} \, dx = \int \frac{x^4 + 3x^3 + 6x^2 + 6x + 5}{(x+1)(x^2+2)} \, dx \)

(b) \( \int_{\frac{1}{2}}^{2\sqrt{3}} \frac{1}{\sqrt{16 - x^2}} \, dx \)

(c) \( \int \frac{x^2}{\sqrt{16 - x^2}} \, dx \)

(d) \( \int_{\frac{\sqrt{2}}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} \, dx \)

3. [25 Points] For each of the following improper integrals, determine whether it converges or diverges. If it converges, find its value.

(a) \( \int_{6}^{\infty} \frac{1}{x^2 - 10x + 28} \, dx \)

(b) \( \int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - x^2} \cdot \arcsin x} \, dx \)

(c) \( \int_{1}^{\infty} \frac{1}{x^2 + 5x + 6} \, dx \)

(d) \( \int_{0}^{1} x \ln x \, dx \)

4. [15 Points] Find the sum of each of the following series (which do converge):

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n 7^n}{3n-1} \)

(b) \( \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{n+1} (\ln 5)^n}{n!} \)

(c) \( \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n-1}}{3(2n)!} \)

(d) \( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots \)

(e) \( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots \)

5. [35 Points] In each case determine whether the given series is absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n (n^3 + 7)}{n^7 + 3} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^n \arctan(7n)}{n^7 + 7} \)

(c) \( \sum_{n=1}^{\infty} n \cdot \arcsin \left( \frac{1}{n} \right) \)

(d) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{3n} (2n)!}{n^4 (n!)^2} \)

(e) \( \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2 + 4} \)
6. [15 Points] Find the **Interval** and **Radius** of Convergence for the power series
\[
\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n) (4x - 1)^n}{n^2 \cdot 5^n}.
\]
Analyze carefully and with full justification.

7. [8 Points]
(a) Write the MacLaurin Series for \( f(x) = x^4 \arctan(2x) \). State the Radius of Convergence for this series.

(b) Use this series to determine the **seventh**, **eighth** and **ninth** derivatives of \( f(x) = x^4 \arctan(2x) \) evaluated at \( x = 0 \). Do **Not** Simplify your answers here in part (b).

8. [12 Points] Please analyze with detail and justify carefully. Simplify your answers.

(a) Estimate \( e^{-\frac{1}{3}} \) with error less than \( \frac{1}{100} \). Justify in words that your error is indeed less than \( \frac{1}{100} \).

(b) Estimate \( \arctan \left( \frac{1}{2} \right) \) with error less than \( \frac{1}{100} \). Justify in words that your error is indeed less than \( \frac{1}{100} \).

(c) Estimate \( \cos(1) \) with error less than \( \frac{1}{10} \). Justify in words that your error is indeed less than \( \frac{1}{10} \).

9. [15 Points]
(a) Consider the region bounded by \( y = e^x - 1, \ y = 3, \ x = 0 \). Rotate the region about the vertical line \( x = -1 \). **Set-Up** but **DO NOT EVALUATE** the integral representing the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

(b) Consider the region bounded by \( y = \arcsin x, \ y = 1, \) and \( x = 0 \). Rotate the region about the vertical line \( x = 5 \). **Set-Up** but **DO NOT EVALUATE** the integral representing the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

(c) Consider the region bounded by \( y = \arctan x, \ y = 4, \ x = 0 \) and \( x = 1 \). Rotate the region about the **\( y \)-axis** **COMPUTE** the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

10. [15 Points]
(a) Consider the Parametric Curve represented by \( x = (\arctan t) - t \) and \( y = 2 \sinh^{-1} t \).
**COMPUTE** the **arclength** of this parametric curve for \( 0 \leq t \leq \sqrt{3} \).
Recall \( \frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1 + x^2}} \)

(b) Consider a different Parametric Curve represented by \( x = t + \frac{1}{t} \) and \( y = \ln (t^2) \). **COMPUTE** the **surface area** obtained by rotating this curve about the **\( y \)-axis** for \( 1 \leq t \leq 2 \).

11. [15 Points] Compute the **area** bounded outside the polar curve \( r = 1 + \sin \theta \) and inside the polar curve \( r = 3 \sin \theta \). **Sketch** the Polar curves and shade the bounded area.