Stat 111
Exam 3

100 minutes

Date: _____

Name: _____

You may use a calculator and three (one-sided) pages of 8.5" by 11" notes, which you will turn in with your exam. Please show all your work, including all calculations, and explain your answers for partial credit.

Please write responses to all questions in the space provided. We tried to provide more space than necessary. For most of the questions, we suggest using 1-2 sentences for your answer. Long explanations will generally not earn extra points and will slow you down.

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Problem	Points
Basics (20 pts)	
Hands Dirty (19 pts)	
Hypothesis Testing (20 pts)	
Regression (20 pts)	
Incentives (11 pts)	

Total (90 points)

I Back to Basics (20 pts)

1. What is the symbol for a Standard Normal random variable? (2 pts)

   (a) X (b) x (c) Z (d) z (e) t

2. For each scenario, identify and categorize the explanatory and the response variables, then choose the appropriate display. (3 pts each)

   (a) Is there a relationship between gender and test score (e.g., do males and females score differently on the same test)?

   Explanatory variable:_____ Categorical or Quantitative

   Response variable:_____ Categorical or Quantitative

   Side-by-side boxplots

   Scatterplot Histogram

   Two-way table Pie chart

   (b) Is there a relationship between gender and smoking status?

   Explanatory variable:_____ Categorical or Quantitative

   Response variable:_____ Categorical or Quantitative

   Side-by-side boxplots

   Scatterplot Histogram

   Two-way table Pie chart

   (c) Can we use SAT score to predict freshman GPA?

   Explanatory variable:_____ Categorical or Quantitative

   Response variable:_____ Categorical or Quantitative

   Side-by-side boxplots

   Scatterplot Histogram

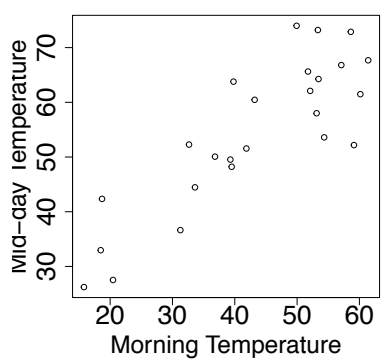
   Two-way table Pie chart

3. Questions on the correlation coefficient, r. Correct all errors with each of the following (3 pts each):

(a) "Adding 100 points to every person's SAT score in Question 2(c) will cause r to increase."

(b) "There is zero correlation between my effort and my grades. It seems like when I work harder, my grades go down."

(c) The scatterplot between the daily low temperatures and the highs has $r = 0.8°F$.



II Getting Hands Dirty (19 pts)

An environmental scientist claims that more than 25% of American households plan on installing solar panels. To test her claim she needs to survey a sample of households.

1. Would you advise her to take a random sample or a convenience sample? Briefly explain the benefit and the downside of your recommendation. (4 pts)

2. What type of random sampling design would you implement to survey the households in the U.S.? Explain. (2 pts)

3. You realize that Amherst College has students from all over the U.S. (and from abroad), so you could help with the query by asking students to be a representative of their respective households. You look up Facebook, randomly sample 144 students, and request their households' plans on the installation of solar panels. What is the sampling frame that you used? What type of bias, if any, would this sample have? Explain. (3 pts)

4. Assume that the sample you collect is representative of the U.S. population. The proportion of "yes" is $\frac{1}{3}$ in the sample. Check the appropriate conditions and construct a two-sided 95% confidence interval for the proportion of households which are positively considering the installaion of solar panels. (4 pts)

5. What would you tell the investigator about her claim? Would you have given a different answer at the 98% confidence level, and why? (4 pts)

6. With a 100% response rate from the fellow Amherst students, you decide to take another random sample of 144 students. You, then, merge the first and the second sample and construct a 95% confidence interval. Without doing any calculation describe the changes you expect to see from the previous to new confidence intervals. (2 pts)

III Hypotheses Testing (20 pts)

Refer to a reduced Z-table here when necessary:

Confidence level	z^*
90 %	1.645
92.5%	1.780
95 %	2
97.5%	2.241

Refer to a T-table here when necessary:

Significance level = α

Two-tail		0.40	0.20	0.10	0.02	0.010	0.002	0.001
One-tail	(df)	0.20	0.10	0.05	0.01	0.005	0.001	0.0005
	1	1.38	3.08	6.31	31.82	63.66	159.15	318.31
	2	1.06	1.89	2.92	6.96	9.92	15.76	22.33
	3	0.98	1.64	2.35	4.54	5.84	8.05	10.21
	4	0.94	1.53	2.13	3.75	4.60	5.95	7.17
	5	0.92	1.48	2.02	3.36	4.03	5.03	5.89
	6	0.91	1.44	1.94	3.14	3.71	4.52	5.21
	7	0.90	1.41	1.89	3.00	3.50	4.21	4.79
	8	0.89	1.40	1.86	2.90	3.36	3.99	4.50
	9	0.88	1.38	1.83	2.82	3.25	3.83	4.30
	10	0.88	1.37	1.81	2.76	3.17	3.72	4.14
	11	0.88	1.36	1.80	2.72	3.11	3.62	4.02
	12	0.87	1.36	1.78	2.68	3.05	3.55	3.93
	13	0.87	1.35	1.77	2.65	3.01	3.49	3.85
	14	0.87	1.35	1.76	2.62	2.98	3.44	3.79
	15	0.87	1.34	1.75	2.60	2.95	3.39	3.73
	16	0.86	1.34	1.75	2.58	2.92	3.36	3.69
	17	0.86	1.33	1.74	2.57	2.90	3.33	3.65
	18	0.86	1.33	1.73	2.55	2.88	3.30	3.61
	19	0.86	1.33	1.73	2.54	2.86	3.27	3.58
	20	0.86	1.33	1.72	2.53	2.85	3.25	3.55
	21	0.86	1.32	1.72	2.52	2.83	3.23	3.53
	22	0.86	1.32	1.72	2.51	2.82	3.21	3.50
	23	0.86	1.32	1.71	2.50	2.81	3.20	3.48
	24	0.86	1.32	1.71	2.49	2.80	3.18	3.47
	25	0.86	1.32	1.71	2.49	2.79	3.17	3.45
	26	0.86	1.31	1.71	2.48	2.78	3.16	3.43
	27	0.86	1.31	1.70	2.47	2.77	3.15	3.42
	28	0.85	1.31	1.70	2.47	2.76	3.14	3.41
	29	0.85	1.31	1.70	2.46	2.76	3.13	3.40
	30	0.85	1.31	1.70	2.46	2.75	3.12	3.39
	Large	0.84	1.28	1.64	2.33	2.58	2.88	3.09

1. In a hypothesis test for $H_0 : \mu = 0.04$ vs. $H_A : \mu \neq 0.04$, it was found from a sample of 36 that $\bar{x}=$ 0.06, and it was known that $\sigma = 0.02$. Calculate the test statistic for this test. (3 pts)

   (a) 1 (b) 0 (c) 0.06 (d) 6 (e) 0.04

2. In a hypothesis test for $H_0 : \mu = 3$ against the alternative $H_A : \mu \neq 3$, in a sample of 14 people (the experimental units) a t_{test}-statistic of 3.816 was found. What is the range for the P-value in this test? (3 pts)

   (a) P-value < 0.001
   (b) $0.001 < P$-value < 0.002
   (c) $0.002 < P$-value <0.005
   (d) $0.005 < P$-value < 0.01
   (e) $0.01 < P$-value < 0.05

3. A car manufacturer is interested in the gas mileage of all cars when additive is used. So, it took a random sample of 25 cars with additive and found a sample mean of 19.00 mpg (miles per gallon) with a standard deviation of $7.50mpg$. When additive is not used, cars have a mean gas mileage of $18.25mpg$. Does using additive improve gas mileage?

   (a) Define the parameter of interest in words. (2 pts)

   (b) Set up the null and the alternative hypotheses using symbols. (3 pts)

   (c) Calculate the test statistic and use an appropriate symbol. (3 pts)

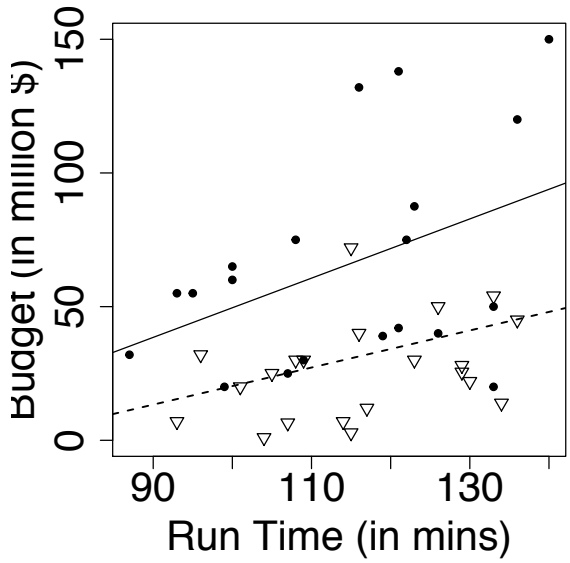
(d) What is the P-value based on the test statistic? (2 pts)

(e) Regardless of your answer in (d), use the P-value of 0.2 and state the formal conclusion of the hypotheses test at the significance level of 0.02. (2 pts)

(f) State the conclusion in context. (2 pts)

IV Movies (20 points)

How does the cost of a movie depend on its length and genre? Here is a scatterplot of the cost (millions of dollars) versus the running time (in minutes) for Action movies (solid circle) and Drama movies (upside-down triangle).




```
> summary(Budget[Genre==``Action''']/1000000)
   Min.  1st Qu.  Median    Mean   3rd Qu.    Max.     sd
   20.00   37.25   55.00   65.53   78.12    150.00   40.40

> summary(RunTime[Genre==``Action''])
  Min.  1st Qu.  Median   Mean   3rd Qu.     Max      sd
  87.0   100.0   117.5   114.4   123.8      140.0    15.48
```

Below is an R output for two sets of regression of the cost of movie making on the run time. The top is for Action movies released in 2011 and the bottom is for Drama's in 2011.

```
Coefficients:                Estimate Std. Error t value Pr(>|t|)
(Intercept)                   -61.200     64.274  -0.952   0.3536
RunTime[Genre == ``Action"]     1.108      0.557   1.989   0.0622 .
Residual standard error: 37.58 on 18 degrees of freedom
Multiple R-squared:  0.1801,Adjusted R-squared:  0.1346
F-statistic: 3.955 on 1 and 18 DF,  p-value: 0.06215
```

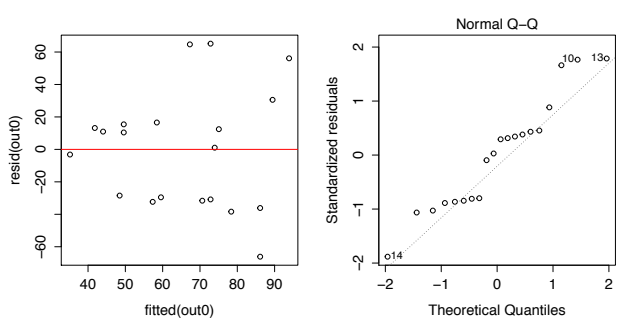
```
Coefficients:                Estimate Std. Error t value Pr(>|t|)
(Intercept)                 -49.0687    30.3409  -1.617  0.11790
RunTime[Genre == ``Drama"]    0.6935     0.2413   2.874  0.00797 **

Residual standard error: 28.94 on 26 degrees of freedom
Multiple R-squared:  0.2411,Adjusted R-squared:  0.2119
F-statistic:  8.26 on 1 and 26 DF,  p-value: 0.007973
```

1. Write the regression equation for Action Movies. Then, describe it in words. (4 pts)

2. An action movie director is introducing a new movie, and I heard through the grapevine that the movie runs for 180 minutes. We are interested in estimating the total budget for her movie. Use the regression equation to predict the budget. (2 pts)

3. Using the scatterplot above and the diagnostic plots below, check the assumptions for a simple linear regression. Are you concerned of any violation of the assumptions? (6 pts)



4. Assume that all necessary conditions are met. Describe the shape of the confidence band for the fitted line of the action movies in words and add the confidence band to the the scatterplot on page 10. (3 pts)

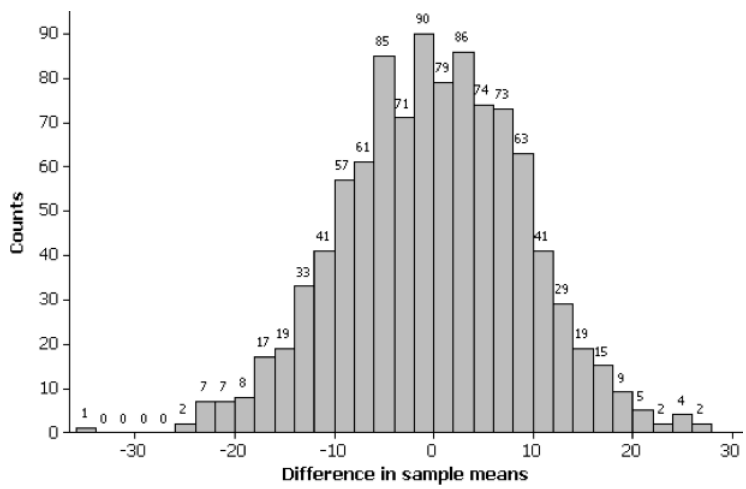
5. Make a reasonable argument about your prediciton of the budget for this director's new release. Would you prefer to provide an interval for your prediction or a single value? Why? You may use an equation or allude to the meaning of the word "regression". (5 pts)

V Incentives : When do we perform the Best? (11 points)

A psychology student conducted a research study in which the research question is whether financial incentives can improve performance on video games. The student prepares an experiment in which 40 subjects are randomly assigned to one of two groups. The first group was offered \$5 for a score above 100 and the other group was simply told to "do your best." Each of the 40 students played the video game and achieved a certain score, from which the student calculated the following summary statistics:

		\$5 incentive	"Do your best"
Mean Score		98	80
Sample Size		20	20

These average scores differ by $98 - 80 = 18$ points, but the student is concerned that this difference might be just due to random chance. A student simulated a sampling distrubution of this difference under the null. She graphed the 1000 statistics (differences in group means) to produce the following histogram:



Use this information to circle the best answer among the options to the following questions.

1. What is the sampling distribution about?

   (a) It allows her to compare her actual result to what could have happened by chance if gamers' performances were not affected by whether they were asked to do their best or offered an incentive.

   (b) It allows her to determine the percentage of time the \$5 incentive strategy would outperform the "do your best" strategy for all possible scenarios.

   (c) It allows her to determine how many times she needs to replicate the experiment for valid results.

   (d) It allows her to determine whether the normal distribution fits the data.

2. What is the appropriate condition the sampling distribution assumes, i.e. what is the null scenario?

   (a) The \$5 incentive is more effective than the "do your best" incentive for improving performance.
   (b) The \$5 incentive and the "do your best" incentive are equally effective at improving performance.
   (c) The "do your best" incentive is more effective than a \$5 incentive for improving performance.
   (d) Both (a) and (b) but not (c).

3. Which of the following could be the approximate P-value in this situation? Recall that the research question is "Does the \$5 incentive improve performance?"

   (a) 0.501 (b) 0.047 (c) 0.022 (d) 0.001

4. What is your conclusion about whether \$5 incentives are effective in improving performance on the video game?

   (a) The \$5 incentive is not effective because the sampling distribution of differences is centered at zero.
   (b) The \$5 incentive is effective because the sampling distribution of differences is centered at zero.
   (c) The \$5 incentive is not effective because the P-value is greater than 0.05.
   (d) The \$5 incentive is effective because the p-value is less than 0.05.

5. Which of the following is the appropriate interpretation of the P-value?

   (a) The P-value is the probability that the \$5 incentive is not really helpful.
   (b) The P-value is the probability that the \$5 incentive is really helpful.
   (c) The P-value is the probability that she would get a result at least as extreme as the one she actually found, if the \$5 incentive is really not helpful.
   (d) The P-value is the probability that a student wins on the video game when presented with the \$5 incentive.