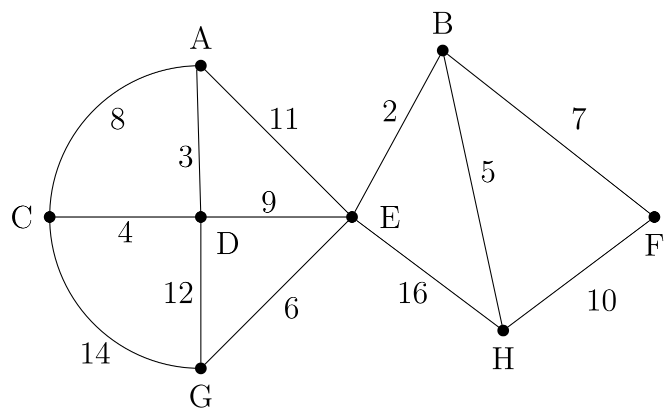
Math 220: Discrete Mathematics
Final Exam, May 9, 2017

Attempt all problems. **Show all of your work**.
No notes, textbooks, calculators or outside help may be used on this exam.

1. (10 points)

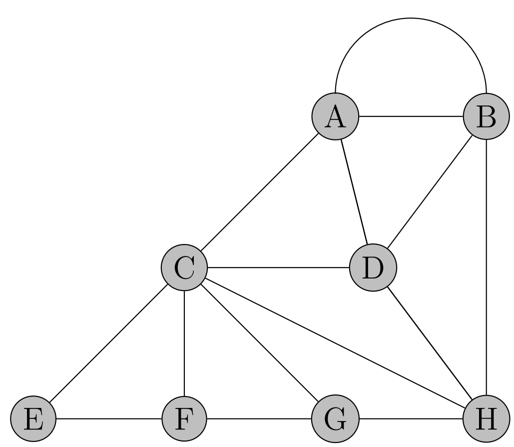
   (a) Use Kruskal's Algorithm to find a minimum cost spanning tree on the graph shown below. Mark the edges selected and compute the cost of the resulting network.



   (b) Explain how we know that the graph above does not contain any Eulerian trail.

2. (10 points)

   (a) Find an Eulerian trail on the graph below by numbering the edges in the order that you travel across them, and marking each edge with an arrow indicating the direction of travel.



(b) Find a Hamiltonian circuit on this graph and describe it by listing the vertices in the order that they are visited.

3. (8 points) A school district with 4 schools receives 8 identical whiteboards, to be divided among the schools as they see fit. How many ways can they be distributed if:

   (a) there are no restrictions and any school can receive any number of whiteboards?

   (b) each school must receive at least 1 whiteboard?

   (c) each school must receive exactly 2 whiteboards?

4. (8 points) Let $f : A \to B$ and $g : B \to C$ be functions.

   (a) Prove: If $g \circ f$ is injective, then f is injective.

   (b) Disprove: If $g \circ f$ is surjective, then f is surjective.

5. (10 points) Let $f : A \to B$ be a one-to-one function, and let X and Y be subsets of A. Prove that $f(X \cap Y) = f(X) \cap f(Y)$.

6. (6 points) Recall that a relation R on A is a subset of $A \times A$. Show that the intersection $R \cap S$ of two equivalence relations R and S on a set A is also an equivalence relation.

7. (10 points) Recall that a standard 52 deck of cards can be divided into 4 suits $(\heartsuit, \diamondsuit, \clubsuit, \spadesuit)$ and into 13 denominations $(2 - 10, J, Q, K, A)$. Find the probability that a hand of 13 cards contains:

   (a) an Ace and King of the same suit

   (b) 4 cards of the same denomination

   (c) one card of each denomination

   (d) at least 2 cards that have the same denomination

8. (6 points) Show that $3 + 11 + 19 + \cdots + (8n - 5) = 4n^2 - n$ for all $n \in \mathbb{N}$.

9. (5 points) Show that $4|(3^{2n} + 7)$ for all $n \in \mathbb{N}$.

10. (10 points) Consider the following statements:

    i. $P \Rightarrow (Q \vee R)$

    ii. $(P \wedge \sim R) \Rightarrow Q$

    (a) Construct a truth table to show that the two statements are equivalent.

(b) Give the contrapositive of statement (ii), .

11. (5 points) Give the negation of the following statement:
$\forall \epsilon > 0, \exists \delta > 0$ such that $|x| < \delta \Rightarrow |f(x)| < \epsilon$

12. (12 points) Let D be the set of all positive divisors of 20.

(a) List the elements of D.

(b) Let $\cong$ be the equivalence relation on D such that $x \cong y$ if x and y have the same first letter when written out as a word. Find the associated partition of S into equivalence classes.

(c) Draw a Hasse diagram for the partial order relation "divides" on D.

(d) Your Hasse diagram in part (c) is a graph. What is the adjacency matrix for this graph? (When ordering the vertices for the matrix, go from lowest to highest number in D.)