1. [10 points] Evaluate each limit. Justify your answer.

   (a) $\displaystyle\lim_{x\to 0} \frac{x^3}{\sin x - \arctan x}$

   (b) $\displaystyle\lim_{x\to \frac{1}{2}} (1+\ln(2x))^{\tan(\pi x)}$

2. [15 points] Evaluate each integral.

   (a) $\displaystyle\int_0^{\pi/2} \cos^4 t \, dt$

   (b) $\displaystyle\int_0^{\sqrt{2}} \frac{1}{(x^2+2)^{5/2}} \, dx$

   (c) $\displaystyle\int \frac{\arctan x}{x^2} \, dx$

3. [15 points] For each improper integral, evaluate it if possible, or determine that it diverges.

   (a) $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x}+\sqrt{x}} \, dx$

   (b) $\displaystyle\int_0^{\infty} \frac{x}{x^2+3x+2} \, dx$

   (c) $\displaystyle\int_e^{\infty} \frac{1}{x\left((\ln x)^2+\ln x\right)} \, dx$

4. [15 points] Evaluate each of the following sums (you may assume that each sum converges).

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$ (express your answer in terms of x).

   (b) $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \cdots$

   (c) $\displaystyle\frac{1}{2} - \frac{1}{2^2\cdot 2} + \frac{1}{2^3\cdot 3} - \frac{1}{2^4\cdot 4} + \cdots$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{n!}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n+1)!} x^n$, assuming that $x>0$ (express your answer in terms of x).

5. [20 points] For each series, determine whether it is **absolutely convergent, conditionally convergent**, or **divergent.** Justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n+\sqrt{n}}{n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n!}{(2n)^n}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+100n}{3^n}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{\arctan(n)}{n^2}$

6. [7 points] Find the **interval of convergence** for the following power series. Analyze carefully with full justification.

$$\sum_{n=0}^{\infty} \frac{1}{n^2+n+1} x^n$$

7. [4 points] Suppose that a patient takes a single dose of a certain medication every day for a long period of time. Each dose is $100mg$ of medication, which is slowly removed from the patient's bloodstream according to the following rule: t days after the patient takes the dose, the number of milligrams left from that dose in the patient's bloodstream is $100 \cdot \left(\frac{4}{5}\right)^t$.

   If the patient continues to take this medication once a day for a very long period of time, how many milligrams of medication will be in her bloodstream immediately after taking a dose?

8. [8 points] Consider the region bounded by the curves $y = e^x$, $y = e$, and the y-axis. This region is revolved around the line $x = -1$ to obtain a solid. Compute the volume of this solid.

9. [10 points] Consider the paramteric curve given by the following equations.

$$\begin{aligned} x(t) &= 4\sinh(t) \\ y(t) &= 8\cosh(t) \end{aligned}$$

   (a) Determine the tangent line to this curve when $t = \ln 2$.

   (b) Set up but **do not evaluate** an integral computing the **arc length** of this curve between $t = 0$ and $t = \ln 2$.

   (c) Set up but **do not evaluate** an integral for the **surface area** obtained by revolving this curve around the y-axis.

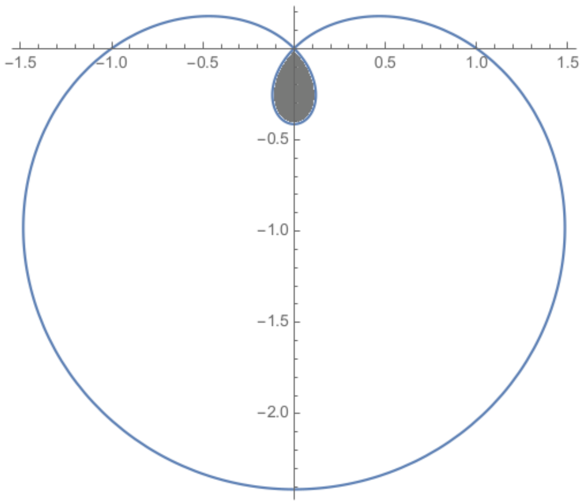
10. [12 points] For each region described, set up but **do not evaluate** an integral (or sum of integrals) for its area.

   (a) The area bounded inside the polar curve $r = 2 - 2\cos\theta$.

   (b) The area outside the polar curve $r = 1$ and inside the polar curve $r = 1 - \sin\theta$.

   (c) The area inside both of the polar curves $r = 1$ and $r = 2 - 2\cos\theta$.

   (d) The image below depicts the polar curve $r = 1 - \sqrt{2}\sin\theta$. Set up **but do not evaluate** an integral computing the area of the shaded region (enclosed by the "inner loop" of the curve).



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