NAME ______________________________

PROFESSOR GABRIEL SOSA

INSTRUCTIONS

1. There are 12 problems and a total of 15 pages (including this cover sheet).
2. Read the problems carefully and answer ONLY what is being specifically asked, in the way that it is being asked.
3. To receive any credit, you must show sufficient work. Partial credit will be given ONLY if sufficient progress towards an answer is shown. No credit will be given to correct answers obtained from inconsistent and/or incorrect work.
4. Simplify all your answers.
5. Place all final answers in the boxes provided, and express them in the form requested/specified.
6. You must comply with Amherst's Honor Code and the guidelines provided on the syllabus regarding examinations.
7. **No Calculators are allowed**

Good luck!

Problem	Points	Max. Possible
1		8
2		16
3		16
4		16
5		16
6		16
7		16
8		16
9		16
10		16
11		16
12		16
13		16
TOTAL		200

1. (8 points) I was fortunate to have you this semester as a student. Take a deep breath. Relax! This final test is a formality to signify the culmination of a journey; but the things you have learned will stay with you. That is what's important. Now, write: "I am ready! I can do this!".

2. (16 points) Consider the planes $\mathfrak{P}_1$, $(x - z - 1 = 0)$, and $\mathfrak{P}_2$, $(y + 2z - 3 = 0)$.
   (a) (8 points) Find the vector equation of the line $\mathfrak{L}$ where $\mathfrak{P}_1$ and $\mathfrak{P}_2$ intersect.

(b) (8 points) Find the linear equation of the plane perpendicular to the plane $\mathfrak{P}_3$, $(x + y - 2z - 1 = 0)$, and containing $\mathfrak{L}$.

3. (16 points) The trajectories of two particles are given by

$$\overrightarrow{r_1}(t) = \langle t, t^2, t^3 \rangle$$

and

$$\overrightarrow{r_2}(t) = \langle -1 + 3t, 1 + 3t, -1 + 9t \rangle.$$

(a) (6 points) Do the two particles collide? Explain.

(b) (10 points) Find all the points where the two trajectories intersect.

4. (16 points) Consider a particle moving in the space. The particle's position at time t is given by the vector function $\overrightarrow{r(t)} = \langle e^{2t}, 2e^t, t\rangle$

   (a) (10 points) Determine the distance traveled by the particle after t seconds $\overrightarrow{r(t)}$.

   (b) (6 points) Determine a_T, the tangential component of the acceleration vector $\overrightarrow{a(t)}$ of the particle.

5. (16 points) Consider the function

$$f(x,y) = \begin{cases} \dfrac{3xy(y^2-x^2)}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) (6 points) Compute $f_x(0,0)$ and $f_y(0,0)$.

(b) (10 points) Show that $f(x,y)$ is not differentiable at $(0,0)$.

6. (16 points) Find the linear equation of the tangent plane to the surface

$$xe^{z+1} + yx^2 - \frac{z}{y} = 3$$

at the point $(-2, 1, -1)$.

7. (16 points) Find the critical points of the function $f(x, y) = x^3 - 3xy + y^3$ and classify each of them as a maximum, minimum, or saddle point.

8. (16 points) Consider the solid bounded by $0 \leq y \leq x$, $x^2 + y^2 \leq 4$ and $0 \leq z \leq xy$.

   (a) (5 points) Compute the volume of this solid.

(b) (5 points) Compute the area of the top part of this solid.

(c) (6 points) Compute the lateral area of this solid.

9. (16 points) Use the change of variables $x = v$ and $y = u^2 + v$ to compute the integral

$$\iint_R \frac{e^{\sqrt{y-x}}}{x+2-y} dA$$

where R is the triangle bounded by $x \geq 0$ and $x \leq y \leq 2$.

10. (16 points) Using spherical coordinates, set up, but (**do not evaluate**), the formula needed to compute the mass of the solid bounded by $\dfrac{x^2+y^2}{3} \leq z^2 \leq x^2+y^2$ and the sphere $x^2+y^2+z^2 \leq 8$, if its density is given by the function $\delta(x,y,z) = \dfrac{x}{z}$.

11. (16 points) Consider $\overrightarrow{F(x,y)} = \langle e^x \sin y + 3y^2, e^x \cos y + 6yx - 2x \rangle$

 (a) Is this vector field conservative? Justify your answer.

 (b) Compute $\int_C \overrightarrow{F} \bullet d\overrightarrow{r}$, where C is the parabola $y = x^2$ starting at $(0,0)$ and ending at $(1,1)$.

12. (16 points) Let C be the boundary of the triangle with vertices $(0,2)$, $(2,2)$ and $(0,-2)$ oriented *clockwise*. Compute the line integral

$$\int_C (\sin 3x^2 + y)dx \, + \, x^3 dy.$$

Extra credit (10 points):

Prove that the gradient $\nabla f(a, b)$ of a function $f(x, y)$ is perpendicular to the level curve of f that passes through (a, b) if the partial derivatives of the function are continuous everywhere.