1. (10 points)
(a) (5 points) Find an equation of the plane \( P \) which passes through the line of intersection of the two planes \( x - z = 1 \) and \( y + 2z = 3 \) and is also perpendicular to the plane \( x + y - 2z = 1 \).
(b) (5 points) Find the distance between the skew lines \( x = y = z \) and \( 1 + x = \frac{1}{2}y = \frac{1}{3}z \).

2. (10 points)
(a) (5 points) Let \( C \) be the curve of intersection of the parabolic cylinder \( x^2 = 2y \) and the surface \( 3z = 2xy \). Compute the curvature of the curve \( C \) at \( (1, \frac{1}{2}, \frac{1}{3}) \).
(b) (5 points) Find the arc length of the segment of the curve \( \vec{r}(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle \) between \( (1, 0, 1) \) and \( (-e^\pi, 0, e^\pi) \).

3. (10 points)
(a) (5 points) We are given a partial derivative of a function \( f(x, y) \):
\[
 f_y(x, y) = \begin{cases} 
 0 & \text{if } (x, y) = (0, 0), \\
 \frac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2} & \text{if } (x, y) \neq (0, 0), 
\end{cases}
\]
Compute \( f_{yx}(0, 0) \) and \( f_{yy}(0, 0) \).
(b) (5 points) Find a unit vector \( \vec{u} \) such that the directional derivative of \( f(x, y) = x \ln y + x \) at the point \( (1, 1) \) in the direction of \( \vec{u} \) is equal to 0.

4. (10 points)
(a) (4 points) Is the function \( f(x, y) = \begin{cases} 
 \frac{xy^3}{x^2 + y^6} & \text{if } (x, y) \neq (0, 0) \\
 \frac{1}{2} & \text{if } (x, y) = (0, 0)
\end{cases} \) continuous at \( (0, 0) \)? Prove your claim.
(b) (6 points) Consider the vector field \( \vec{F} = \langle y, x, z \rangle \).
   i. (3 points) Is \( \vec{F} \) conservative? Explain.
   ii. (3 points) Is there another vector field \( \vec{G} \) on \( \mathbb{R}^3 \) such that \( \vec{F} \) is the curl of \( \vec{G} \)? Explain.
5. (10 points)
(a) (5 points) Find the critical points of the function
\[ f(x, y) = e^{xy^2} - x \]
and decide whether each critical point is a local maximum, local minimum or saddle point.
(b) (5 points) Find the absolute maximum and minimum values of the function
\[ f(x, y) = x + 2y \]
subject to the constraint \( x^2 + y^2 \leq 4 \).

6. (10 points)
(a) (5 points) Sketch the curve \( r = \sqrt{2} \cos(2\theta) \) and compute the area of the region it bounds.
(b) (5 points) Let \( I = \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \). Find the value of \( I \).

\text{Hint: Compute } I^2 \text{ first.}

7. (10 points)
(a) (5 points) Find the volume of the solid bounded by the surfaces \( x = 0, x = y^2, z = 2y, z = 4 \) and \( y = 0 \).
(b) (5 points) Find the volume of the solid bounded above by the sphere \( x^2 + y^2 + z^2 = 1 \) and below by the cone \( z = \sqrt{3}(x^2 + y^2) \).

8. (10 points)
(a) (5 points) Use Green’s Theorem to compute the area of the region bounded by the curve
\[ \vec{r}(t) = \left( \frac{\cos t + \sin t}{3}, \frac{2 \sin t - \cos t}{3} \right), \]
where \( t \) goes from 0 to \( 2\pi \).
(b) Use the change of variables
\[ u = x + y, v = 2x - y \]
to find the area of the region on the \( x - y \) plane bounded by the curve
\[ (x + y)^2 + (2x - y)^2 = 1. \]
9. (10 points)

(a) (5 points) Compute the surface integral \( \iint_S ydS \) where \( S \) is the part of the cylinder \( y^2 + z^2 = 4 \) which is between the planes \( x = 0 \) and \( x + y = 4 \).

(b) (5 points) Use Stokes’ Theorem to evaluate \( \iint_S (\nabla \times \vec{F}) \cdot d\vec{S} \) where \( \vec{F}(x, y, z) = \langle -y, x, xyz \rangle \) and \( S \) is the part of the sphere \( x^2 + y^2 + z^2 = 25 \) that lies below the plane \( z = 4 \), oriented so that the unit normal vector at \( (0, 0, -5) \) is \( \langle 0, 0, 1 \rangle \).

10. (10 points)

(a) (5 points) Let \( \vec{F}(x, y, z) = \langle e^{yz} \cos z, x^2 z, z \rangle \) and \( S \) be the surface of the cube whose vertices are \((±1, ±1, ±1)\), oriented with outward normals. Compute the surface integral

\[
\iint_S \vec{F} \cdot d\vec{S}.
\]

*Hint:* Divergence Theorem.

(b) (5 points) Let \( D \) be a simply-connected closed planar region on the \( x-z \) plane. The solid \( E \) is defined by \( \{(x, y, z) \in \mathbb{R}^3 | (x, z) \in D, g_1(x, z) \leq y \leq g_2(x, z)\} \) where \( g_1 \) and \( g_2 \) are two smooth functions defined on \( D \). If \( Q(x, y, z) \) is a real-valued smooth function defined on \( \mathbb{R}^3 \), prove that

\[
\iint_{\partial E} \vec{Q} \cdot d\vec{S} = \iiint_E Q_y dV,
\]

where the boundary surface \( \partial E \) is given the outward orientation. You cannot use the divergence theorem, because you are actually proving part of it.