The New Keynesian Price Puzzle

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Abstract

The “price puzzle”—a short-run increase in prices in response to higher interest rates—is a common feature of estimated responses to monetary policy shocks. In this paper, I show that it is also a robust prediction of the New Keynesian model: nearly all equilibria consistent with common identifying assumptions and estimates of interest rate responses result in a price puzzle. Although these results imply that *exogenously* higher interest rates can increase inflation in the short run, the model nevertheless always recommends raising rates *endogenously* to lower inflation after non-monetary shocks. This result calls into question the practice of inferring causal effects of endogenous policy changes from responses to exogenous policy shocks.

Keywords: Monetary policy, fiscal policy.

JEL classification:

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1 Introduction

The middle column of Figure 1 displays some prominent examples of what Eichenbaum (1992) termed the "price puzzle": an increase in the price level after a monetary policy shock that increases the short term nominal interest rate. It is considered a puzzle because it goes against the common intuition that higher nominal interest rates should lower inflation, not raise it.

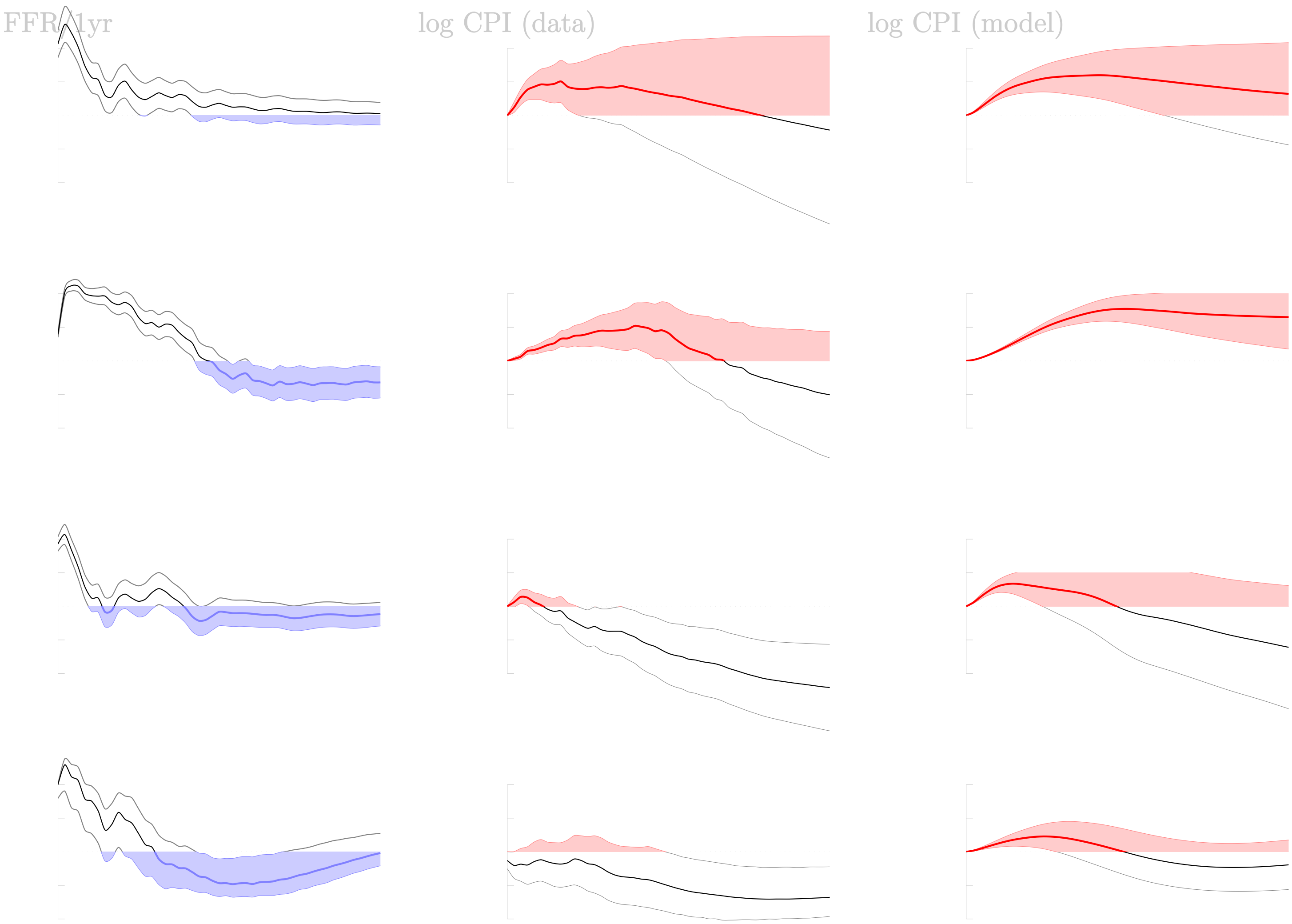
A number of theoretical explanations of and empirical fixes to the price puzzle have been put forward. In this paper I provide a new one: the price puzzle is a robust prediction of the standard New Keynesian model. It is only in certain knife-edge cases of a first-order autoregressive disturbance to a monetary policy rule that the model does *not* exhibit a price puzzle. But the left column of Figure 1 suggests that the response of interest rates typically does not follow an AR(1) process and in fact frequently turns negative after a contractionary monetary policy shock. Shocks to monetary policy in this model that more closely resemble their empirical counterparts almost always result in a price puzzle. This fact is most readily seen in the right column of Figure 1, which displays model responses to the estimated responses of the interest rate along with the assumption that initial inflation is zero.[1] Given the estimated interest rate response, the model predicts price puzzles in line with the estimated price level responses.

The mechanism is in part due to the forward-looking nature of the model: output and inflation are present discounted values of future variables, including the nominal interest rate. Inflation cannot fall unless the interest rate eventually falls as well. A quick visual examination of Figure 1 bears this out: the more blue in the left column, the less red in the middle and right columns. The price puzzle, if the New Keynesian model's logic is to be believed, occurs because in the present value calculation, the initial rise in the interest rate offsets the expected future declines that will occur in response to the ensuing recession. It happens in the data; it happens in the model.

Does this resolution of the price puzzle mean that central banks should lower interest rates to stave off inflation? No. This question illustrates the limitations of extrapolating the estimated effects of exogenous shocks to endogenous policy responses. The required policy responses to inflationary shocks conform to the usual logic: raise rates to bring down inflation.

[1] The zero-inflation response is an identifying restriction in two of the displayed responses; it is imposed on estimated responses in Romer and Romer (2004), and relaxed in Gertler and Karadi (2015). I describe the construction of these equilibria in Section 3, below.

Figure 1: Examples of the price puzzle



Response of the short-term interest rate (left column, Federal Funds rate in rows 1–3, one-year Treasury in row 4) and the log consumer price index (estimates in the middle column, model responses in the right column) to contractionary monetary policy shocks. First row: Stock and Watson (2001); second row: Romer and Romer (2004); third row: Ramey's (2016) monthly version of Christiano et al. (1999); forth row: Gertler and Karadi (2015). Red areas in the left column indicate *increases* in the price level in response to contractionary monetary policy shocks; blue areas in the right column indicate *decreases* in the short-term interest rate in response to contractionary monetary policy shocks.

Section 2 below briefly overviews equilibrium formation in the New Keynesian model, with particular attention given to the standard equilibrium with no price puzzle. Section 3 describes a broad classification of equilibria. Section 4 overviews the equilibria in which a price puzzle occurs. Section 5 illustrates the pitfalls of extrapolating the effects of monetary policy shocks to the causal effects of endogenous policy responses to inflation shocks. Section 6 concludes.

2 Equilibria in the New Keynesian model

The non-policy block of the New Keynesian model, as described in Woodford (2003) and Galí (2015) can be written as

$$x_t = (1-\alpha)\mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1}) + u_t^x \tag{1}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t^\pi, \tag{2}$$

where π_t is inflation, x_t is the output gap, and i_t is the nominal interest rate, all expressed in deviations from steady state. Equation (1) is the linearized consumption Euler equation, and (2) is the New Keynesian Phillips Curve. Parameters σ, β, and κ are standard parameters relating to the intertemporal elasticity of substitution, the discount factor, and price stickiness. The standard model has $\alpha = 0$, while $\alpha \in (0, 1]$ corresponds to a "discounted Euler equation," as in McKay et al. (2016). When $\alpha = \beta = 1$, the model collapses to Cochrane's (2018) simple model with a static IS curve.[2]

Without a description of how the nominal interest rate is set, the model is indeterminate. The standard solution for pinning down a unique equilibrium is to specify a monetary policy rule of the form $i_t = \phi\pi_t + u_t^i$. If $\phi > 1$ and nominally explosive equilibra in which $\pi \to \pm\infty$ are ruled out, the model has a unique equilibrium. Uniqueness arises with $\phi > 1$ because *all but one* of the potential equilibria result in runaway inflation or deflation and are therefore eliminated.

King (2000) proposes an alternative equilibrium selection mechanism. In it, the central bank specifies a desired path of the nominal interest rate and threatens to throw the economy into hyperinflation or -deflation if the desired path does not obtain. Specifically, if $\{i_t^*, \pi_t^*\}$ is the desired path of the nominal rate and inflation—the latter consistent with the Phillips curve given $\{i_t^*\}$—then the actual nominal rate i_t is set according to $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ with $\phi > 1$. This approach is more general: the standard equilibrium is just one of many non-explosive equilibria that can be realized.

[2]The model with a static IS curve greatly simplifies much of the algebra while maintaining intuition of the full model.

To see the equivalence, consider a monetary policy disturbance that follows an AR(1) in the simple model with a static IS curve, so $\alpha = \beta = 1$.[3] Substituting out the IS equation gives

$$\mathbb{E}_t \pi_{t+1} = \frac{1}{1+\kappa\sigma}(\pi_t + \kappa\sigma i_t - \kappa u_t^x - u_t^\pi).$$

Supposing $i_t = \phi\pi_t + u_t^i$, with $\phi > 1$ and $u_t^i = \eta u_{t-1}^i + \varepsilon_t^i$, where ε_t is an independent and identically distributed monetary policy shock, and setting $u_t^x = u_t^\pi = 0$ this further simplifies to

$$\mathbb{E}_t \pi_{t+1} = \frac{1+\kappa\sigma\phi}{1+\kappa\sigma}\pi_t + \frac{\kappa\sigma}{1+\kappa\sigma}u_t^i.$$

With $\phi > 1$, the coefficient on π_t is greater than one. Iterating forward gives

$$\pi_t = -\frac{\kappa\sigma}{1+\kappa\sigma\phi}\sum_{j=0}^{\infty}\left(\frac{1+\kappa\sigma}{1+\kappa\sigma\phi}\right)^j \mathbb{E}_t u_{t+j}^i + \lim_{T\to\infty}\left(\frac{1+\kappa\sigma}{1+\kappa\sigma\phi}\right)^T \mathbb{E}_t \pi_{t+T}.$$

As long as the limit goes to zero—which it does when we rule out hyperinflations and -deflations—there is a unique solution for π_t. Since the disturbance follows an AR(1), $\mathbb{E}_t u_{t+j}^i = \eta^j u_t^i$, and

$$\pi_t = -\frac{\kappa\sigma}{1-\eta+\kappa\sigma(\phi-\eta)}u_t^i.$$

This result implies that $\pi_{t+1} = \eta\pi_t$, so inflation "inherits" the AR(1) properties of the disturbance. Since $i_t = \phi\pi_t + u_t^i$,

$$i_t = \left\{1 - \frac{\kappa\sigma\phi}{1-\eta+\kappa\sigma(\phi-\eta)}\right\}u_t^i$$

and so i_t is also an AR(1).[4]

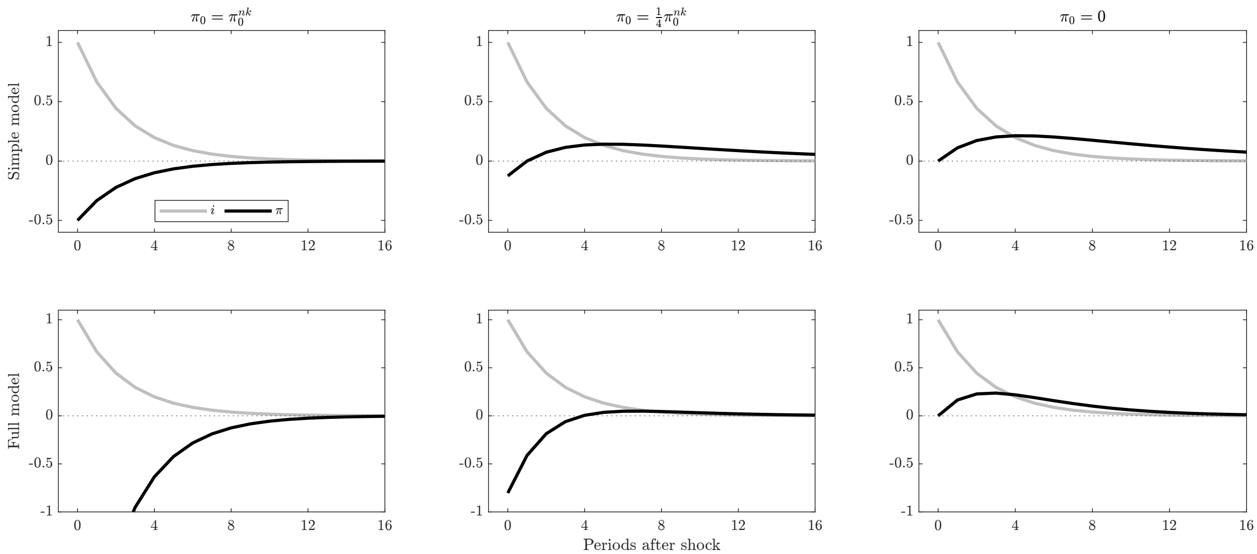
Alternatively, the same equilibrium can be constructed as in King (2000), by specifying a target path for inflation given by $\pi_{t+1} = \eta\pi_t$ and $\pi_0 = -\kappa\sigma/(1-\eta+\kappa\sigma(\phi-\eta))u_0^i$. The impulse responses to a unit shock $u_0^i = 1$ for this equilibrium is displayed in the upper panel of Figure 2. The strength of this method of equilibrium selection is that it makes transparent how to construct others.

As an example, consider *the exact same time path for interest rates* as above, but instead select the equilibrium in which initial inflation is one quarter of its value from the previous equilibrium. Or consider the equilibrium in which initial inflation is zero. These equilibrium

[3]The static IS curve simplifies the math, allows for closed form solutions, and loses very little of the generality. The math for the full model solutions is laid out in the appendix.

[4]A positive shock to ε_t^i, therefore only actually results in an increase in the nominal rate if $\eta(1+\kappa\sigma) < 1$.

Figure 2: Impulse responses to an AR(1) interest rate shock



The response of inflation to a nominal interest rate shock that follows an AR(1) process with $\eta = 2/3$, $\kappa = 0.125$, $\sigma = 1$. First row: $\alpha = \beta = 1$. Second row: $\alpha = 0$, $\beta = 0.95$. Left column: the standard equilibrium; center column: when impact inflation is one quarter that of the standard equilibrium; right column: when impact inflation is zero.

responses are displayed in the second and third columns of Figure 2.

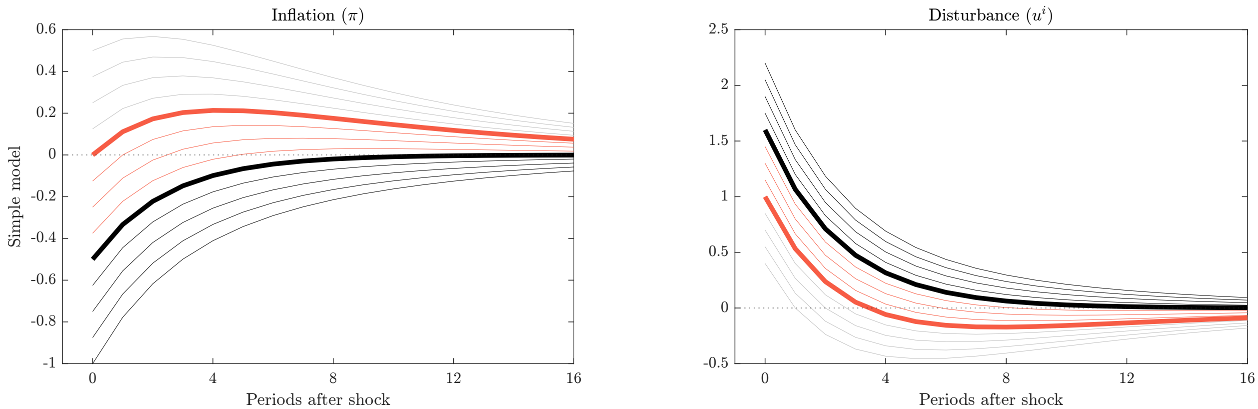
But the standard equilibrium selection technique can also produce these equilibria, just not with an AR(1) disturbance. Let $u_t^i = i_t^* - \phi\pi_t^*$, where $\{i_t^*, \pi_t^*\}$ are the equilibrium interest rate and inflation responses from above. The right panel of Figure 3 displays the required disturbances for these and other equilibria described in Section 3, below. Importantly, *all* are equilibrium responses of inflation for the given disturbance that also result in the same response of the nominal interest rate displayed in Figure 2.

3 Equilibrium classification

The left panel Figure 3 displays the impulse response of inflation corresponding to the path of disturbances displayed in the right panel. As described above, all are equilibrium responses to the same path of the nominal interest rate induced by different disturbances. The inflation responses include those from the left and right columns of Figure 2—the thick black and red lines, respectively—but also a number of others. The equilibria can be classified according to the sign and timing of the responses of inflation to the interest rate shock.

The black lines in Figure 3 are *Keynesian* equilibria: in response to a temporary increase

Figure 3: Implied disturbance u_t^i for various equilibrium responses to an AR(1) interest rate shock



Thin black lines are Keynesian equilibria. Thin red lines are short-run Keynesian, medium-run Fisherian. Thin gray lines are Fisherian equilibria. Thick black and red lines are the standard New Keynesian equilibrium and the "no inflation jump" equilibrium. The right panel shows the implied disturbance term from the monetary policy rule for each equilibrium path of inflation in the simple model: $\alpha = \beta = 1$, $\eta = 2/3$, $\kappa = 0.125$, $\sigma = 1$.

in nominal interest rates, inflation falls and never rises above zero. The gray lines are *Fisherian* equilibria: in response to a temporary increase in nominal interest rates, inflation rises and never falls below zero. The red lines are *short-run Keynesian, medium-run Fisherian* equilibria: inflation falls or is unchanged initially, but rises above zero for a sustained period before returning to zero asymptotically.[5]

None of the equilibria considered so far display the price puzzle pattern; that is, none are *short-run Fisherian, medium-run Keynesian* with inflation at first positive and only later becoming negative. In fact, such an equilibrium cannot exist if i_t follows an AR(1) process, or, more generally, if i_t never falls below zero at any horizon of the impulse response. This fact is most clearly seen in the simple static-IS model.

Assuming again that $u_t^x = u_t^\pi = 0$ at all horizons, iterating the expression for π_{t+1} backwards gives

$$\pi_t = \left(\frac{1}{1+\kappa\sigma}\right)^t \pi_0 + \kappa\sigma \sum_{j=0}^{t-1} \left(\frac{1}{1+\kappa\sigma}\right)^{j+1} i_{t-j-1}.$$

It is obvious that for any nonnegative level of initial inflation, $\pi_t < 0$ requires the sum to be negative, which in turn requires the nominal interest rate to be negative for some periods. If the response of i_t follows an AR(1), that cannot happen.

The left column of Figure 1, however, raises the question of why much of the literature

[5] All nonexplosive equilibria are long-run Fisherian: that is, $i_t = \pi_t$ as $t \to \infty$.

focuses on AR(1) interest rate responses in the first place.[6] It displays the response of the Federal Funds rate for each of the monetary policy shocks displayed in the left column of Figure 1. Although a contractionary monetary policy shock uniformly raises the interest rate on impact, in all cases it is also true that the interest rate falls below zero at later horizons. Moreover, a quick visual comparison of the figures suggests that the *more negative* the interest rate goes at longer horizons, the *less* of a price puzzle there is. This observation motivates the following section.

4 Price puzzle equilibria in the New Keynesian model

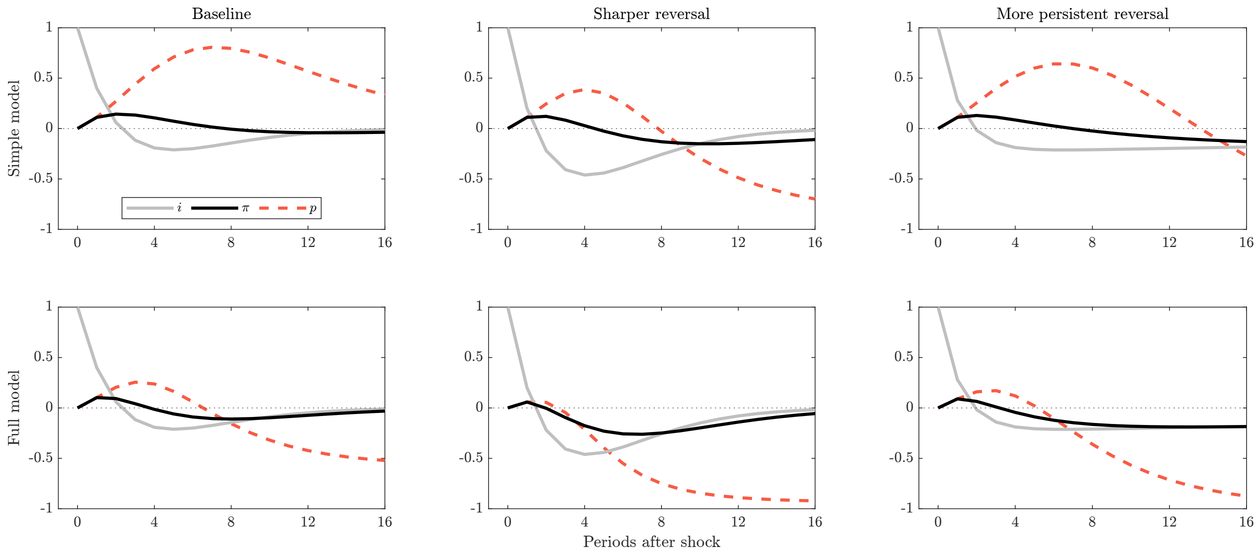
As discussed above, equilibria that exhibit a price puzzle are *short-run Fisherian, medium-run Keynesian.* To illustrate the scenarios in which they occur, I assume that instead of following an AR(1) process, the interest rate follows an ARMA(2,1) process, i.e., $i_t = \eta_1 i_{t-1} + \eta_2 i_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$. This process is flexible enough to result in a similar to shaped response to a one-time shock as those estimated in the the data and displayed in Figure 1. Impulse responses are displayed in Figure 4 for particular values of the coefficients (η_1, η_2, θ) with $\pi_0 = 0$.

The impulse responses displayed are consistent both with the identifying restrictions on the shocks from Figure 1—that interest rates not effect inflation contemporaneously—and with the estimated response of the interest rates—positive on impact and in the first few periods before turning negative. All feature a price puzzle. The price level is shown for comparison to Figure 1.

The baseline case in the left column of Figure 4 simply illustrates that equilibria exist in which a price puzzle occurs. The other columns show how the inflation and price level responses change when the interest rate reversal is quicker (middle column) or more persistent (right column). In each case the integral of the interest rate response is more negative than in the baseline case. They suggest, just as Figure 1 does, that is the long right negative tail of the interest rate response that ultimately drive inflation and the price level lower. The positive initial responses of inflation are the result of the initial higher interest rate outweighing the later negative tail.

[6]Some medium-scale dynamic stochastic general equilibrium models—Christiano et al. (2005) and Smets and Wouters (2007), for example—and models with long-term rates like Gürkaynak et al. (2005) are notable exceptions.

Figure 4: Price puzzle equilbria in the New Keynesian model



Responses of inflation and the price level to ARMA(2,1) interest rate shocks in the no inflation jump equilibrium. *Baseline*: $(\eta_1, \eta_2, \theta) = (1.4, -0.5, -1)$. *Sharper reversal*: $(\eta_1, \eta_2, \theta) = (1.4, -0.5, -1.2)$. *More persistent reversal*: $(\eta_1, \eta_2, \theta) = (1.4, -0.41, -1.125)$. Same calibration as figure 2: $\eta = 2/3$, $\kappa = 0.125$, $\sigma = 1$. First row: $\alpha = \beta = 1$; second row: $\alpha = 0$, $\beta = 0.95$.

5 Endogenous policy responses

The results from the previous section *do not* imply that monetary policymakers should lower interest rates to bring down inflation in response to other shocks. This point highlights the care that is required when applying the estimated effects of monetary policy shocks to causal effects of policy changes made endogenously.

To illustrate this point, consider instead of a monetary policy shock, a "cost-push" shock to the Phillips curve. Let u_t^π follow an AR(1): $u_t^\pi = \rho u_{t-1}^\pi + \varepsilon_t^\pi$, where ε_t^π is i.i.d. Again, consider first the simple model in which $\alpha = \beta = 1$. First, the standard New Keynesian equilibrium with $\phi > 1$:

$$\pi_t = \frac{1}{1+\kappa\sigma\phi} \sum_{j=0}^{\infty} \left(\frac{1+\kappa\sigma}{1+\kappa\sigma\phi}\right)^j \mathbb{E}_t u_{t+j}^\pi = \frac{1}{1-\rho+\kappa\sigma(\phi-\rho)} u_t^\pi.$$

So, inflation again inherits the AR(1) properties of the disturbance; moreover, since $\phi > 1$, the response of inflation is uniformly positive.[7] Unsurprisingly, the standard New Keynesian solution method implies the cost-push shock leads to higher inflation, and an even greater

[7]The same is true in the full model.

increase in interest rates ($i_t = \phi\pi_t$) brings it down, in line with the Taylor principle.

Other possible equilibria include, but are not limited to:

1. a "soft landing" equilibrium, in which $x_t = 0$ at all horizons;

2. a "hard landing" equilibrium, in which $\pi_t = 0$ at all horizons;

3. a "no inflation jump" equilibrium, in which $\pi_0 = 0$, but i_t follows an AR(1);

4. an "arbitrary initial inflation" equilibrium in which i_0 is positive and follows an AR(1), but initial inflation is not specified.

The soft landing equilibrium implies $i_t = \mathbb{E}_t\pi_{t+1}$ at all horizons; plugging in to the simple model gives $\mathbb{E}_t\Delta\pi_{t+1} = -u_t^\pi$. This in turn implies that, if we stipulate that inflation must return to zero in the long run,

$$\pi_t = \sum_{j=0}^{\infty} \mathbb{E}_t u_{t+j}^\pi = \frac{1}{1-\rho} u_t^\pi.$$

So, the soft-landing equilibrium also implies uniformly positive inflation and positive interest rates given by $i_t = \mathbb{E}_t\pi_{t+1} = 1/(1-\rho)\mathbb{E}_t u_{t+1}^\pi = \rho/(1-\rho)u_t^\pi$.

Consider next equilibrium (4), since (2) and (3) end up being special cases of it. Suppose interest rates are chosen to follow an AR(1) $i_t = \nu i_{t-1}$ with i_0 and π_0 chosen arbitrarily. Plugging in and iterating backward gives

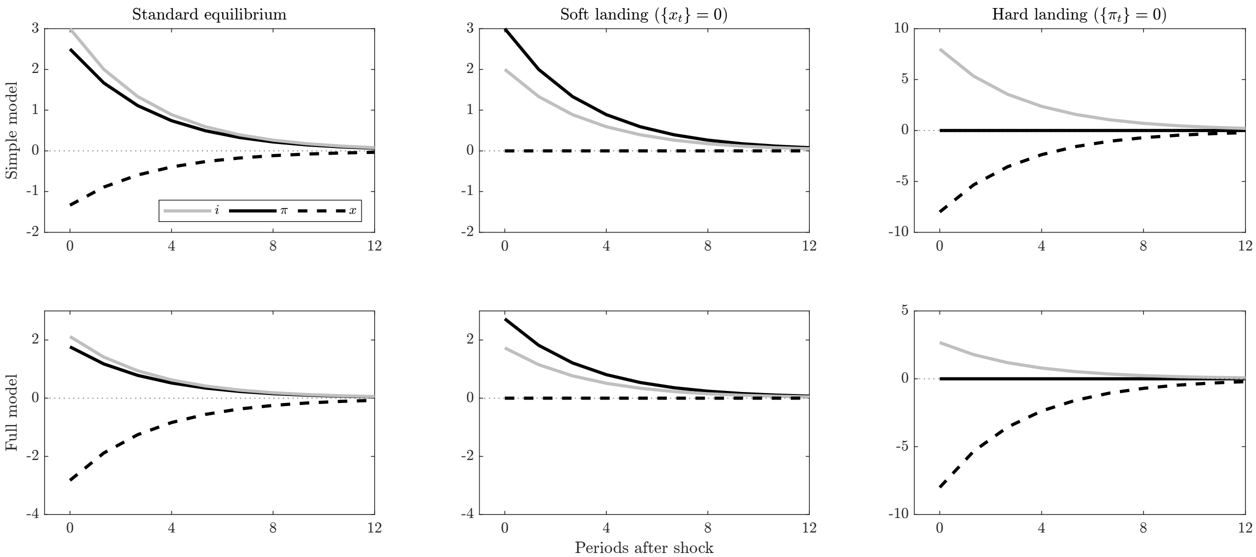
$$\pi_t = \left(\frac{1}{1+\kappa\sigma}\right)^t \pi_0 + \frac{\kappa\sigma}{1+\kappa\sigma}\nu^{t-1}\sum_{j=0}^{t-1}\left(\frac{1}{\nu(1+\kappa\sigma)}\right)^j i_0 - \frac{1}{1+\kappa\sigma}\rho^{t-1}\sum_{j=0}^{t-1}\left(\frac{1}{\rho(1+\kappa\sigma)}\right)^j u_0^\pi.$$

A variety of equilibria encompassed by (3) and (4) can be constructed from the previous expression. But the hard-landing equilibrium (2) is now obvious: if $\pi_0 = 0$, $i_0 = \frac{1}{\kappa\sigma}u_0^\pi$, and $\nu = \rho$, then inflation will be zero at all horizons. This means the initial interest rate is positive and proportional to the cost-push shock, and follows an AR(1) process with the same persistence as u_t^π. In other words, a persistent increase in interest rates is required to bring about no inflation response to the shock. These equilibria are displayed in Figure 5.

Both very hawkish ($\pi_t = 0$) and very dovish ($x_t = 0$) responses to inflation shocks require a persistent increase in interest rates.

These results along with those of the previous section highlight the care needed in translating responses to exogenous monetary policy shocks to the endogenous responses required

Figure 5: Different responses to a cost-push shock



Responses of the interest rate, inflation, and output gap for various equilibrium responses to a cost-push shock.

to respond to other macroeconomic shocks. The behavior of interest rates and inflation after a monetary policy shock depends crucially on equilibrium selection; the required response of interest rates, at least qualitatively, to other shocks does not. Hawks and doves both agree that interest rates need to rise in response to inflation shocks; the question is how much. Whether monetary policy shocks are short-run/medium-run Fisherian/Keynesian has little bearing on the question.

6 Conclusion

The price puzzle is, as Ramey (2016) describes, almost always present in inflation and price-level responses to monetary policy shocks. This paper suggests that the answer to the price puzzle has been hiding in plain sight: it is an equilibrium of the standard New Keynesian model consistent with most restrictions used to identify monetary policy shocks. It does not mean that raising interest rates increases inflation in all cases; that incorrect conclusion results from basing endogenous policy prescriptions on responses to exogenous policy shocks. With the right equilibrium choices, the basic New Keynesian model is consistent with both the price puzzle *and* intuitive policy prescriptions like raising rates to bring inflation down after cost-push shocks. This example raises questions about generalizing the results from other types of policy shocks to the potential effects of endogenous policy changes. I leave this question for future research.

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