Problem 15.5 Diffusion-controlled rates

Solution:

15.30: \[ \text{[m}^3\text{s}^{-1} = \text{[m}^3\text{s}^{-1}][\text{m}] \]

15.31: \[ \text{[m}^3\text{s}^{-1}] = \frac{J}{P_{A2}} = \text{[Nm}^3\text{]} \]

15.32: (same analysis as in 15.31)

(2) \[ k_D = \frac{2k_B T}{3\eta} \left( \frac{r_{AB}}{r_{A2}} \right) = \frac{8.2 \times 10^{-21} J}{3(1.0 \times 10^{-5}\text{Pa} \cdot \text{s})} \left( \frac{1.8\text{nm}}{0.3\text{nm}} \right) \approx 1.6 \times 10^{-17}\text{m}^3\text{s}^{-1} = 9.9 \times 10^9\text{M}^{-1}\text{s}^{-1} \]

(3) LeGow and Parkhurst, Biochemistry 11(24):4520–25 (1972) give the binding rate as 1.5 \( \times 10^7\text{M}^{-1}\text{s}^{-1} \) at 20°C, much slower than the diffusion-controlled rate. Some process besides diffusion slows the binding down by a factor of almost 1000.

Problem 15.6 Restricted Diffusion

Solution:

(1) An “effective” collision is restricted to 0.1 nm x 0.1 nm = 10^2 nm^2 area of myoglobin’s total surface area. \[ A_{AB} = 4\pi(1.8\text{nm})^2 = 40.7\text{nm}^2 \]; this reduces the binding rate by a factor of 2.46 \( \times 10^4 \). The collision-direction angular restriction (10°=0.175 rad) reduces the rate by another factor of

\[ \frac{\pi(0.1 \text{nm})^2}{4\pi x^2} = \frac{0.175^2}{4} = 7.66 \times 10^{-3} \] (\( r \) is an arbitrary distance from an O_2 molecule, used to compute solid angle.) The overall reduction of the binding rate is a factor of 1.88 \( \times 10^6 \). This would make the effective diffusion rate

\[ (1.88 \times 10^{-6})(9.9 \times 10^9\text{M}^{-1}\text{s}^{-1}) = 1.9 \times 10^4\text{M}^{-1}\text{s}^{-1} \], which is now much smaller than the measured rate of about 1.5 \( \times 10^7\text{M}^{-1}\text{s}^{-1} \). This means that restriction of effective collisions to certain areas and in certain directions is a possible explanation of the slower oxygen binding rate, compared to diffusion. The likely explanation of the reduced rate, however, involves a more complex mechanism, with restricted collision, followed by oxygen diffusion through the fluctuating protein structure.
(2) \((1.88 \times 10^{-6})^{-1} = 5.3 \times 10^5\) times.

**Problem 15.7** Binding and protein conformational change.

(1) Solutions:

(2) \(v_{\text{max}} = \frac{L}{\Delta t}\)

(3) \(v_{\text{min}} = \frac{L}{\Delta t} = (0.50 \text{ nm})/(1.0 \times 10^{-12} \text{ s}) = 500 \text{ m/s.}\)

From Figure 14.2, about 40% of the \(O_2\) molecules have this minimum speed.

(4) Assume \(k_D = 10^9 \text{ M}^{-1} \text{ s}^{-1}\). The rate at which \(O_2\) attempts to enter the gate is \([O_2]k_D\).

The concentration of \(O_2\) in water is about 0.3 molcs/m³, or \(3 \times 10^{-4}\) M.

(\text{http://en.wikipedia.org/wiki/File:WOA05\_sea-surf\_O2\_AYool.png})

The attempt rate of \(O_2\) is then \(\text{Attempt rate} = [O_2]k_D = (3 \times 10^{-4}\text{ M}) (10^9 \text{ M}^{-1} \text{ s}^{-1}) = 3 \times 10^5 \text{ s}^{-1}\). If the frequency at which the gate opens is \(f < 3 \times 10^3 \text{ s}^{-1}\), the binding rate will be reduced.

The fraction of time the gate is open is \(\frac{\Delta t}{\tau} = f\Delta t > (3 \times 10^5 \text{ s}^{-1}) (10^{-12} \text{ s}) = 3 \times 10^{-7}\).

**Problem 15.10** Bimolecular Reactions 1

(1) \([B]_0 \gg [A]_0\): \([B(t)] = [B]_0 - ([A]_0 - [A(t)]) = [B]_0 - [A]_0 (1 - e^{-k[3]t})\)

(2)

![Graph 1](image1)

![Graph 2](image2)

Note the vertical axis scales.

(3)

![Graph 3](image3)

![Graph 4](image4)

The log-log plot in this case is not very helpful, since \(B(t)\) changes so little.
Problem 15.11 Bimolecular reactions.2.

\[-\frac{d[A]}{dt} = k[A][B] = k[A][A] - [A]_0 + [B]_0\]

Solution:

(1)

This equation has been dealt with in Chapter 3. See Equation 3.40 and associated discussion.
The guess for a solution should be \( \frac{A(t)}{B(t)} = ce^{-at} \). Following Chapter 3, but reversing A and B,

\( \frac{A(t)}{B(t)} = \frac{A_0}{B_0} e^{-k(B_0 - A_0)} \), with \( B(t) = (B_0 - A_0) + A(t) \)

(2) Solution: Eq. 15.52 is \([A(t)] = [A_0] e^{-k[B]_0} \).

\[\begin{align*}
  \frac{[A]}{[B]} &= \frac{[A]}{[B]}_0 e^{-k[B]_0} \\
  \frac{[A]}{[B]} &= \frac{[A]}{[B]}_0 e^{-k[B]_0} \Rightarrow [A] = [A_0] e^{-k[B]_0}
\end{align*}\]

Eq. 15.55:

(3) Log-log graph:

![Log-log graph](image)

Problem 15.14 Single molecules

Solution: We need to assume a laser beam radius before hitting the lens. We take this radius as typical for a laser, \( R = 1.00 \text{ mm} \).

[Note corrections to early-printing Table 15.5 and section 15.11.1: \( \Delta z = \frac{\lambda I^2}{\pi R^2} \), not \( \frac{\lambda I^2}{\pi R} \):

\( \Delta V = \frac{\lambda I^4}{\pi R^4} \). The second paragraph of section 15.11.1 should be revised as follows: \( \Delta z \) as]
above; concentration 1.0 μM; volume is 1.7 × 10⁻¹⁸ m³; average number of molecules in focal volume is 102; dilute solution by a factor of 100 and one molecule will be in volume.]

(1) See Table 15.5 and Figure 15.28. We use the full longitudinal distance between the two points along the beam axis where the intensity has reduced from I_max to 0.414 I_max less than I_max.

\[
\begin{align*}
\Delta z &= \frac{2\Delta z}{\pi R^2} = \frac{2(420 \times 10^{-2} m)(5.0 \times 10^{-3} m)^2}{\pi(1.0 \times 10^{-3} m)^2} = 6.68 \mu m \\
\Delta V &= \pi R^2 \Delta z = \pi(6.68 \times 10^{-7} m)^2(6.68 \times 10^{-5} m) = 9.36 \times 10^{-14} m^3 = 9.36 \mu m^3 \\
\end{align*}
\]

Note this is a relatively long, cylindrical volume:

(2) \[N = \left(1.0 \times 10^{-14} \text{ moles/L}\right)\left(1000 L/m^3\right)\left(6.02 \times 10^{23} \text{ molee/mole}\right)\left(9.36 \times 10^{-14} m^3\right) = 5.6 \times 10^{-5} \text{ molee} \]

If the concentration is really 10⁻¹⁴ M, this means there is rarely even a single molecule within the focal volume, hardly even two or more molecules simultaneously.

(3) The noise-to-signal ratio is about \(\left(\sqrt{5.6 \times 10^{-5}}\right)^{-1} \approx 100\). How do we interpret this ratio? About 99% of the time there is a signal of zero, occasionally, a signal of \(I_0\) is observed. During this time period, one molecule is within the focal volume. Very rarely would two molecules be in the focal volume. (Note this experiment could be improved—more frequent 1-molecule signals, with only rare 2-molecule—by raising the concentration 10-100X.

(4) The radius of myoglobin is about 1.8 nm, so the diffusion constant is

\[
D = \frac{k_BT}{6\pi\eta R} = \frac{4.1 \times 10^{-21} J}{6\pi(1.0 \times 10^{-5} Pa \cdot s)(1.8 \times 10^{-9} m)} = 1.2 \times 10^{-10} m^2/s. \quad \text{The molecule will most likely diffuse out the side of the cylinder, a distance of 0.668 μm. The time for this to happen is, on average, 0.668 \times 10^{-6} m = (4Dt)^{1/2} = (4 \cdot (1.2 \times 10^{-10} m^2/s) t)^{1/2} \Rightarrow t = 9.3 \times 10^{-4} s \pm 1 ms.}
\]